The large inverse problem of modern radio interferometry

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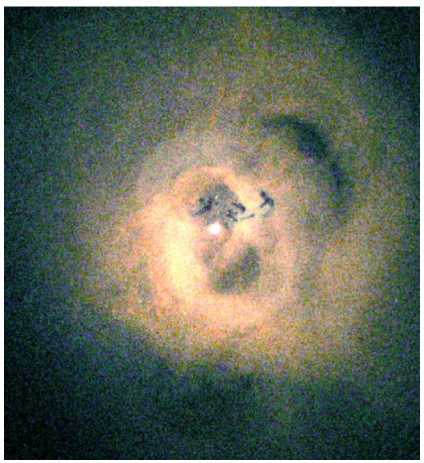
Outline

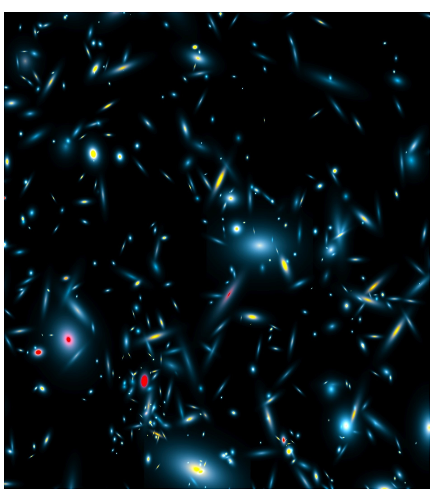
1- Inverting the measurement equation: challenges for the SKA

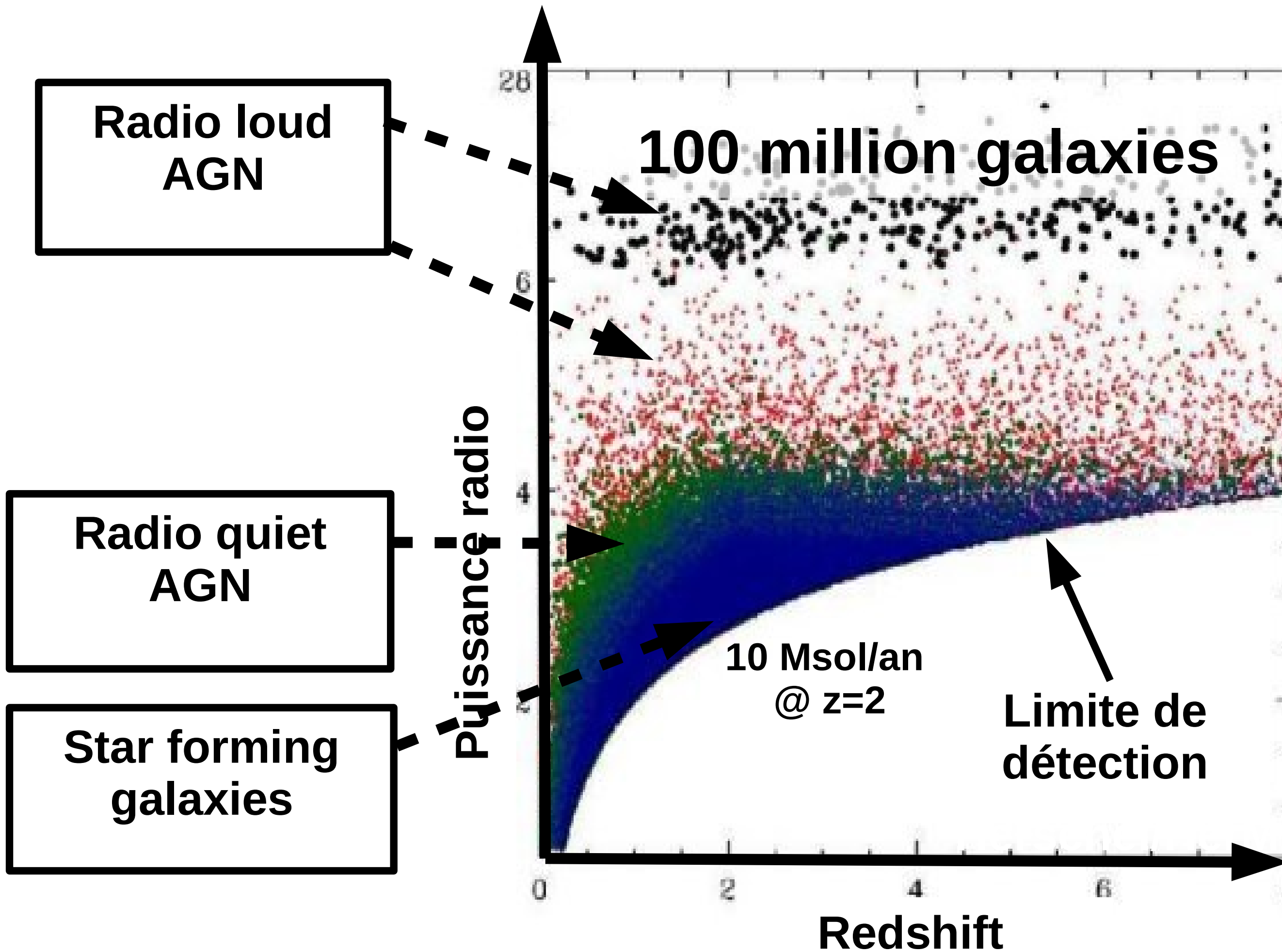
2- Imaging and deconvolution

3- Calibration and ill-conditioning (solvers and filters)

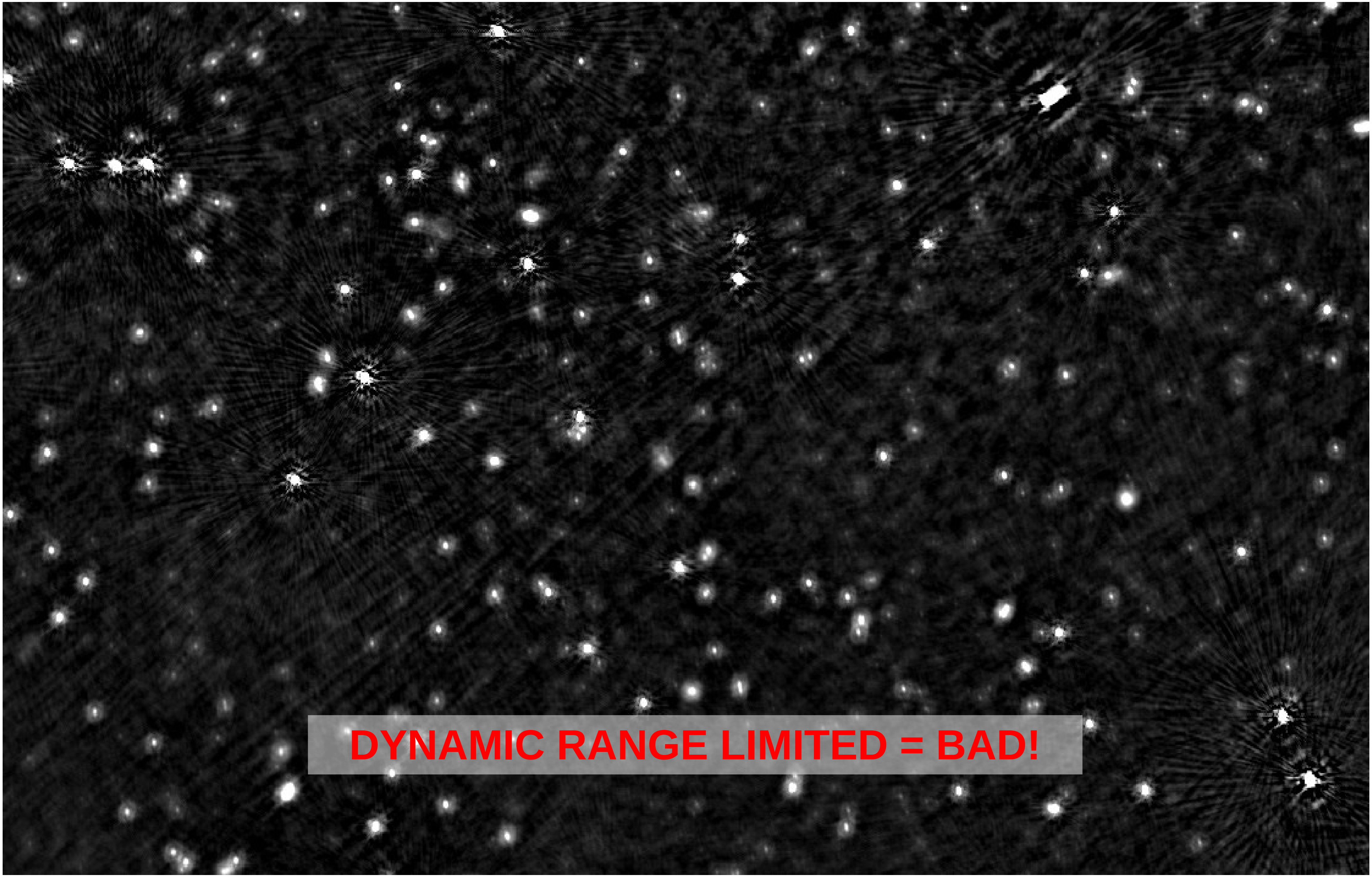
Galaxy formation and AGN evolution



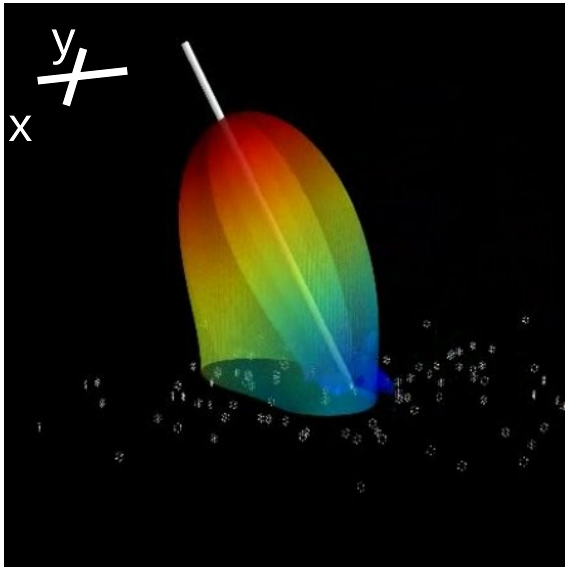


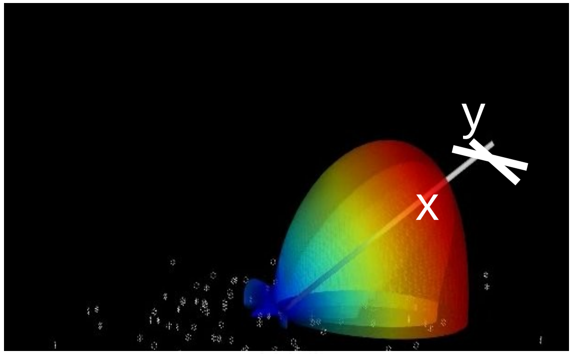


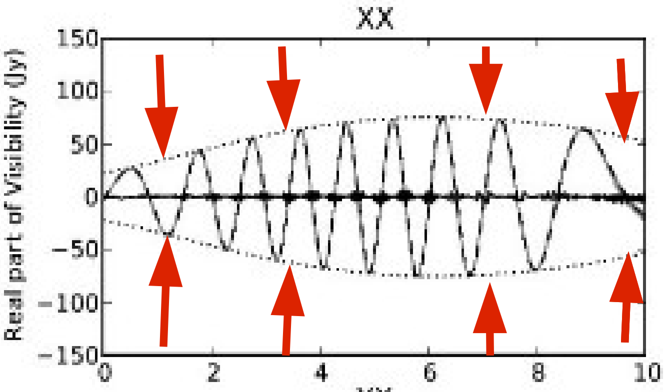
3C295 Observation (110-190 MHz)



… When Direction Dependent Effects (DDE) become a problem : Beam





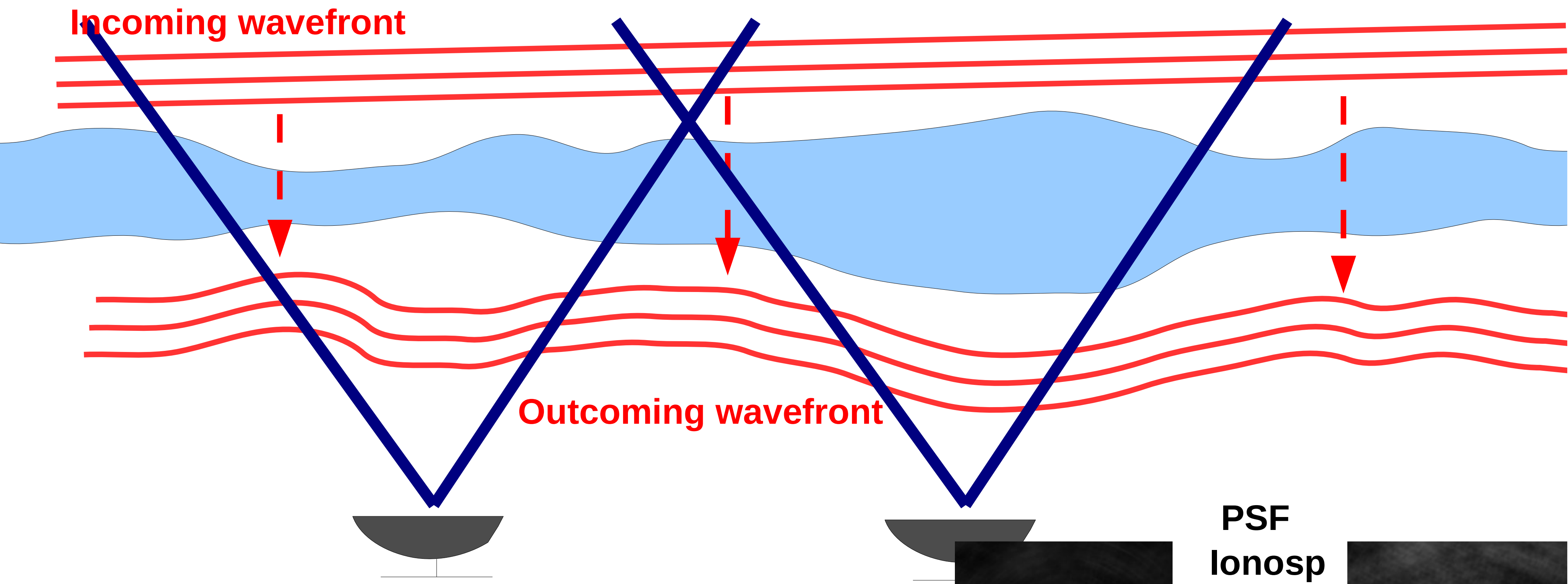


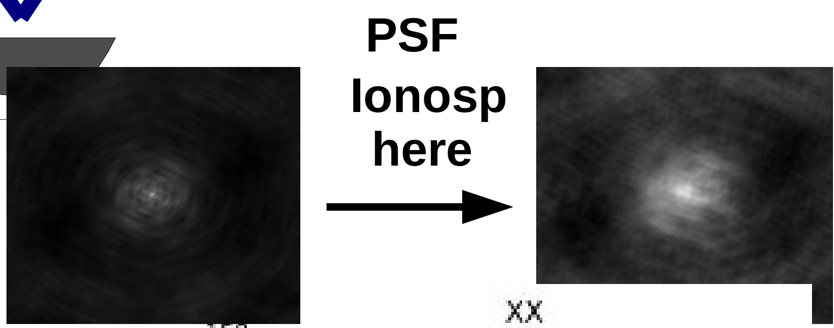
LOFAR stations are phased arrays

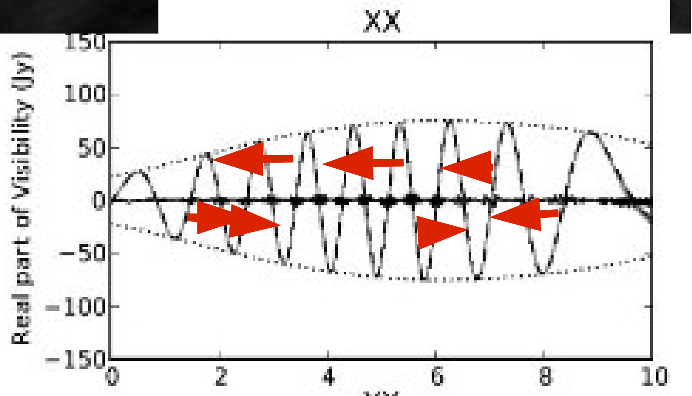
- Beam is variable in frequency and time
- Projection of the dipoles in the sky is non trivial
- Beam can be station-dependent
- Individual clock effects

--> Strong effects on polarisation

… When Direction Dependent Effects (DDE) become a problem : Ionosphere/troposphere







Big field of view : station, direction, time and frequency dependent

Other direction dependent effects :

- Faraday rotation

+ Effect on the polarisation

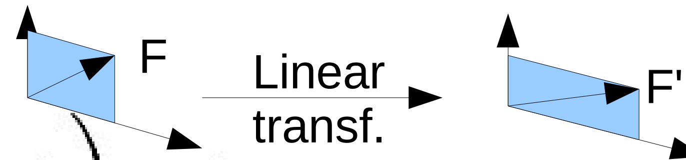
The Measurement Equation

Hamaker 1996

Direction independent

Direction dependent

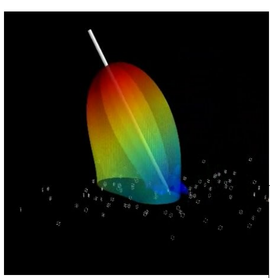
Source coherency



$$V_{pq} = G_p \left(\sum_{i=1}^{N} \mathrm{B}_{pi}\mathrm{K}_{pi}\mathrm{I}_{pi}F_i \; . \; F_i^{+}\mathrm{I}_{qi}^{+}\mathrm{K}_{qi}^{+}\mathrm{B}_{qi}^{+} \right) G_q^{+}$$

[Voltage antenna p] x [Voltage antenna q]*

Beam

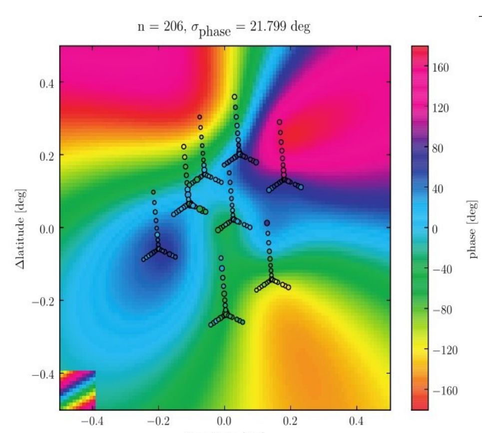


Geometrical delay +Correlator

$$\mathrm{K}_p\mathrm{K}_q^{+} = \exp(-2i\pi\phi_{pq}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

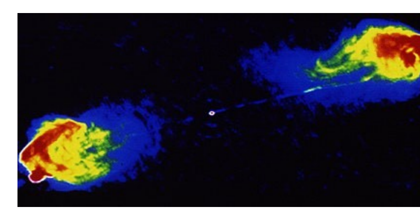
$$\phi_{pq} = u_{pq}l + v_{pq}m + w_{pq}(\sqrt{1 - l^2 - m^2} - 1)$$

Ionosphere



Van der Tol thesis

Electric field



Challenge for the SKA

Measurement (10^13++ points)

$$V_{pq} = G_p \left(\sum_{i=1}^{N} \mathrm{B}_{pi} \mathrm{K}_{pi} \mathrm{I}_{pi} F_i \; . \; F_i^{+} \mathrm{I}_{qi}^{+} \mathrm{K}_{qi}^{+} \mathrm{B}_{qi}^{+} \right) G_q^{+}$$

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Unknown physical process

Unknown Sky

Unknown physical process

Those processes include:

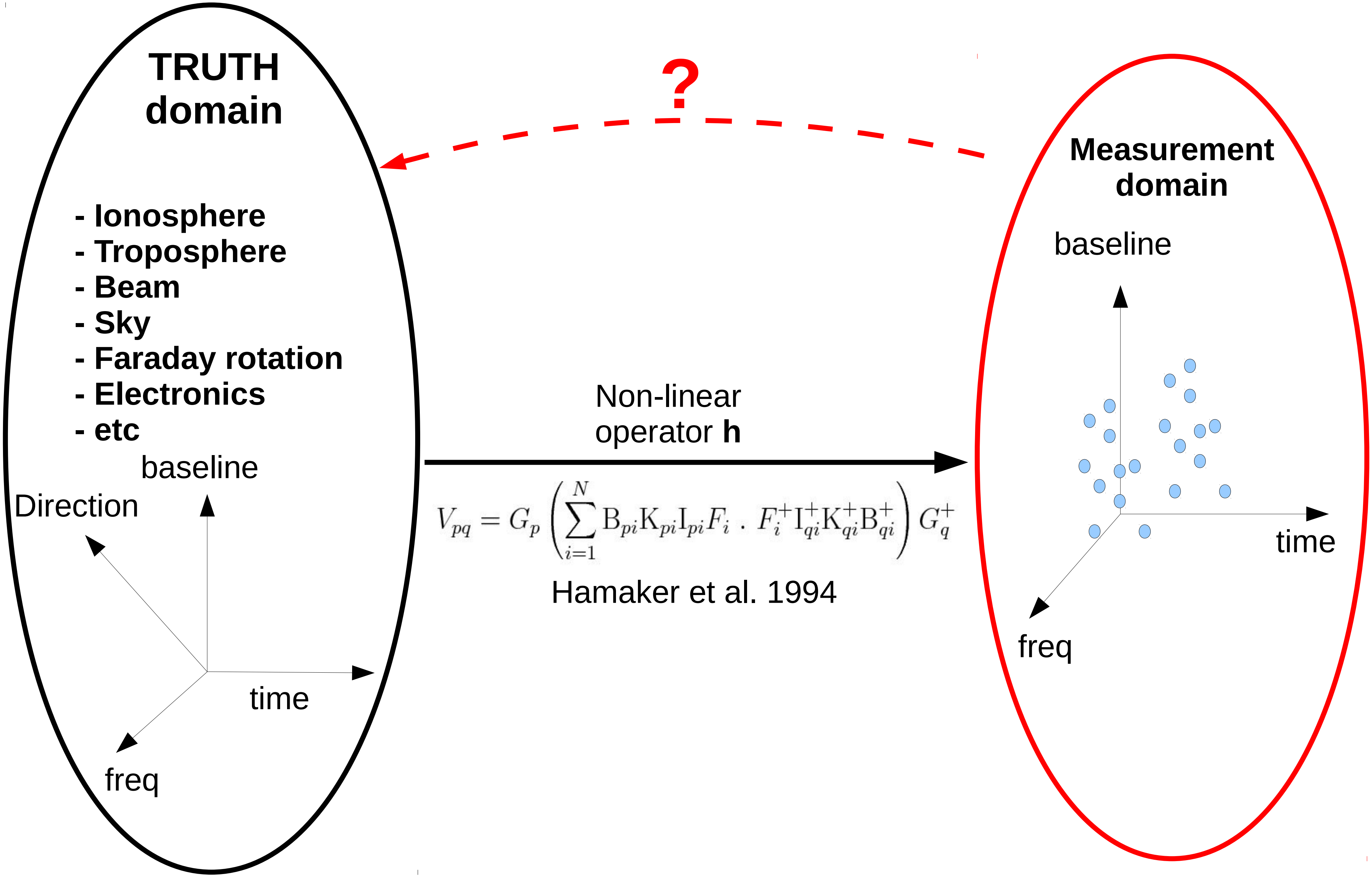
- Antenna beams
- Ionosphere
- Troposphere
- Electronics

But also:

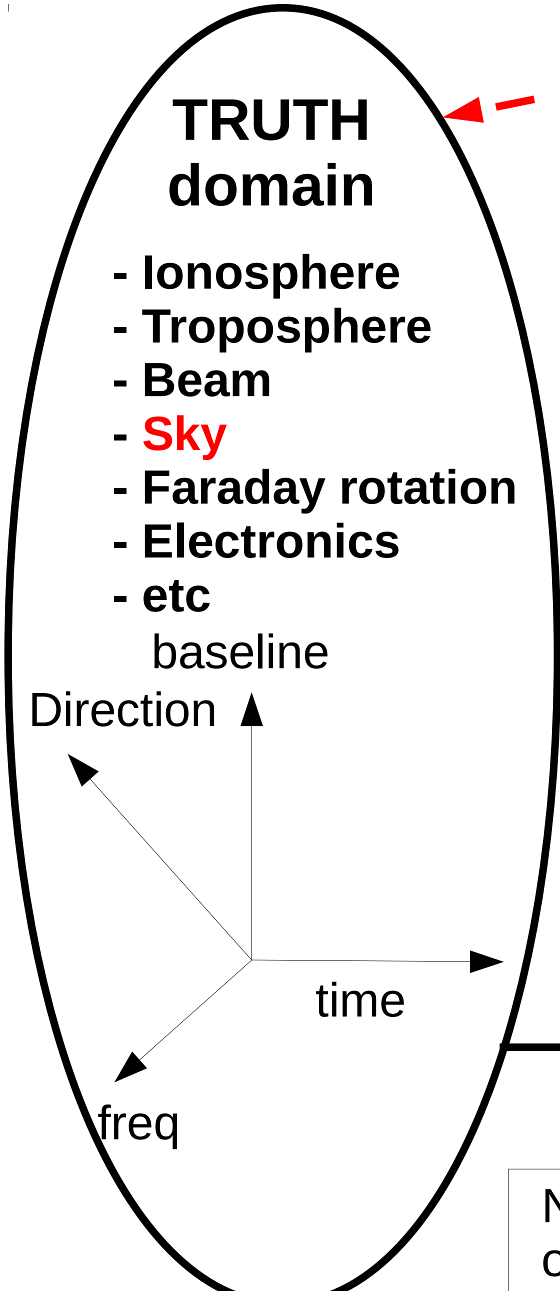
- Solar wind scintillation
- ISM scintillation
- and many more...

→ **Very (very) large inverse (non-linear) problem**

Interferometry

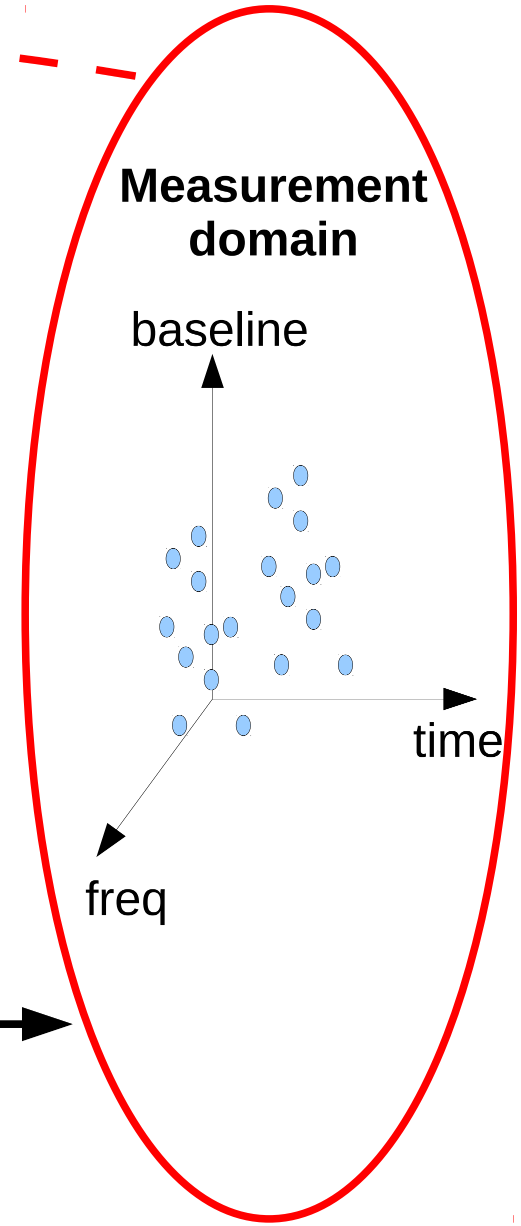


Imaging and deconvolution



Most algorithms use CLEAN:
- Multi-scale (resolved objects)
- Multi-Term (large fractional bandwidth)

New deconvolution algorithms:
- Compressive sensing (understanding and generalisation of CLEAN, see Jean-Luc Starck, and Arwa Dabbech talk)
- Bayesian inference techniques



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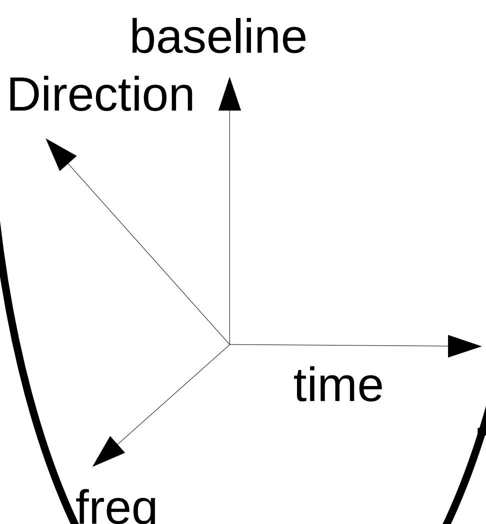
Non-linear operator **h**

Hamaker et al. 1994

Imaging and deconvolution

TRUTH domain

- **Ionosphere**
- **Troposphere**
- **Beam**
- **Sky**
- **Faraday rotation**
- **Electronics**
- **etc**



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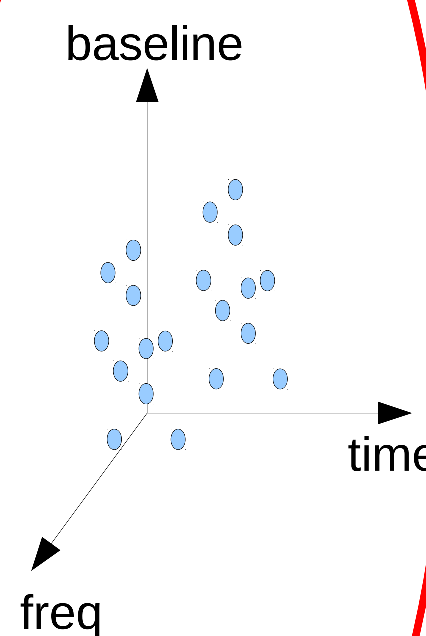
Full polarisation AW-projection (Tasse 12): AWimager for LOFAR

$$V = \mathcal{A}.I$$

Measurement domain



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Full Polarisation A-Projection

Tasse 2013

DDE are in general smooth on the sky --> small support in the uv-domain

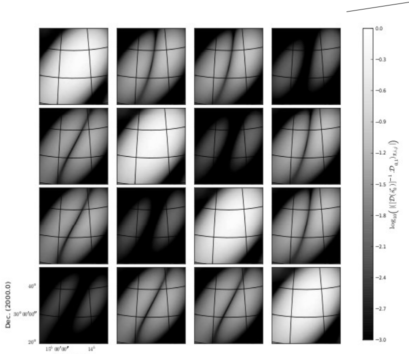
$$\mathrm{Vec}(V_{pq}^{corr}) = \int_S (D_{q,\boldsymbol{s}}^{t\nu,*} \otimes D_{p,\boldsymbol{s}}^{t\nu}).\mathrm{Vec}(I_{\boldsymbol{s}}) . \exp\left(-2i\pi\phi(u,v,w,\boldsymbol{s})\right)d\boldsymbol{s}$$

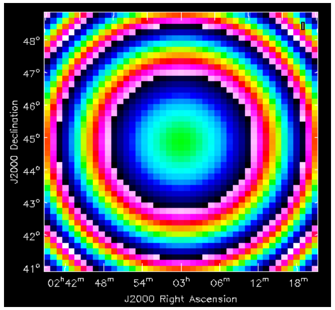
Convolution theorem

$$V_{pq}^{t,\nu}(u,v,w,i) = \mathcal{F}\left(\sum_{j=1}^{4} \mathcal{D}_{pq}^{t,\nu}(i,j,\boldsymbol{s})W(w,\boldsymbol{s})I_j(l,m)\right)$$

$$= \sum_{j=1}^{4}\left[\mathcal{F}\left(\underbrace{\mathcal{D}_{pq}^{t,\nu}(i,j,\boldsymbol{s})}_{\text{DDE (4*4)}}\underbrace{W(w,\boldsymbol{s})}_{\text{W term (scalar)}}\right) * \underbrace{\mathcal{F}\left(I_j(l,m)\right)}_{\text{2D FFT}}\right]$$

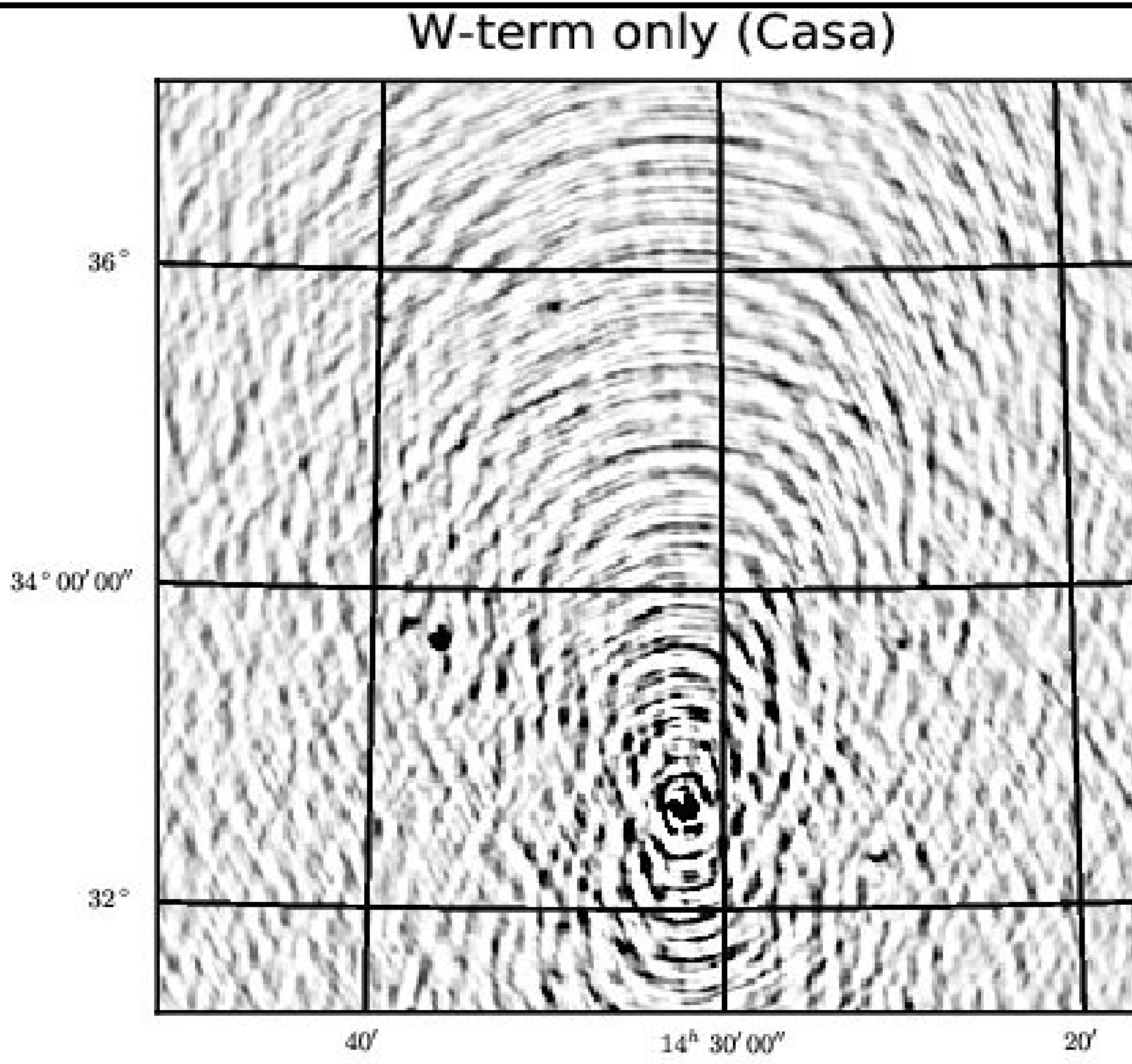
Convolution





Tests on simulated data

100 sources with flux density following NVSS 1.4 GHz source counts



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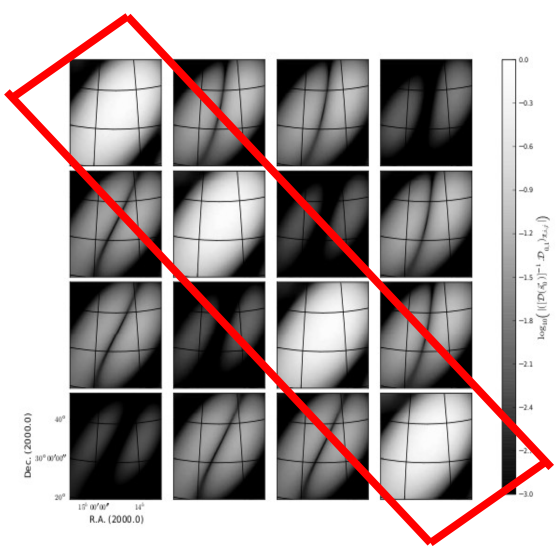
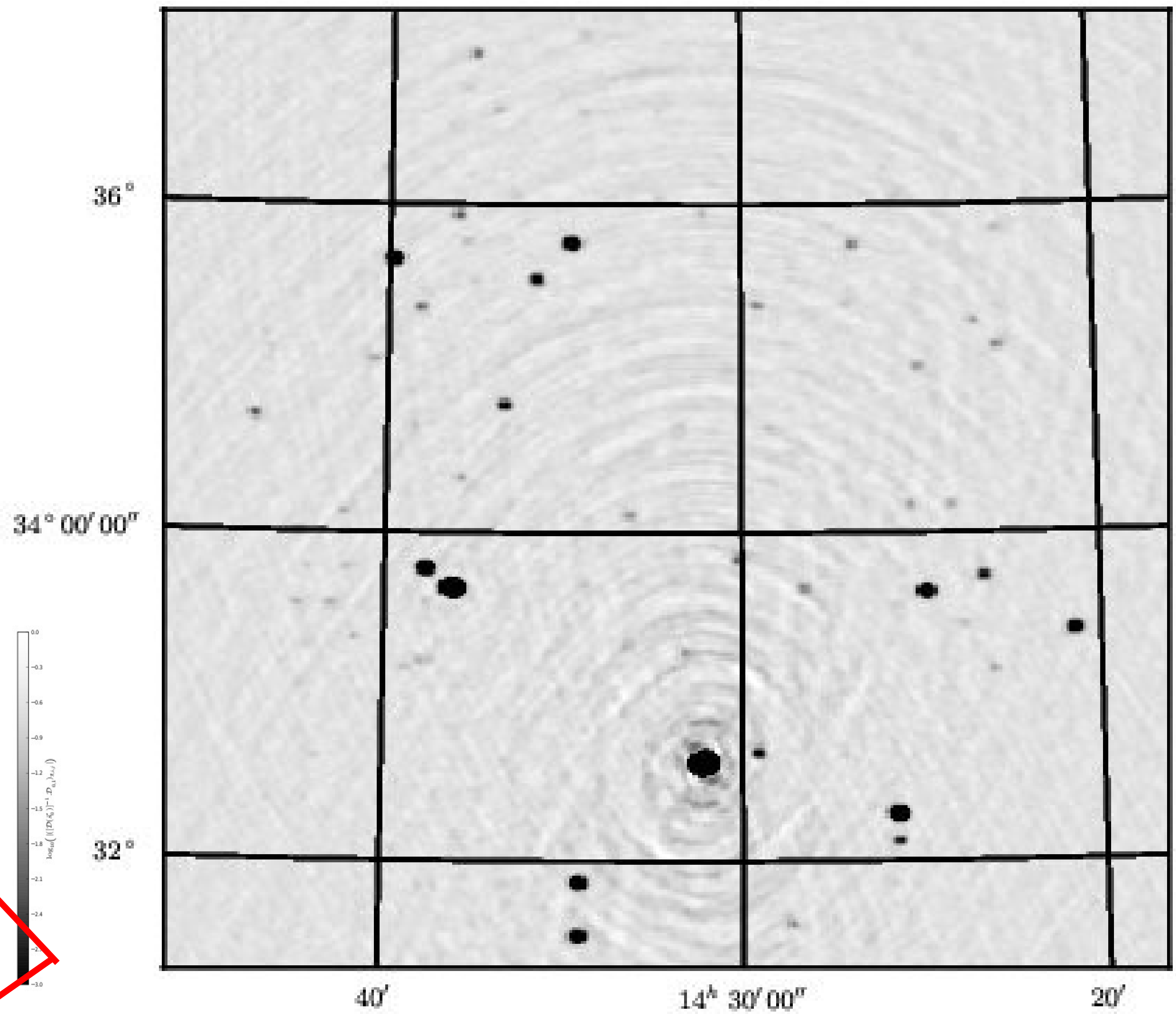


Wterm + Scalar /diagonal beam



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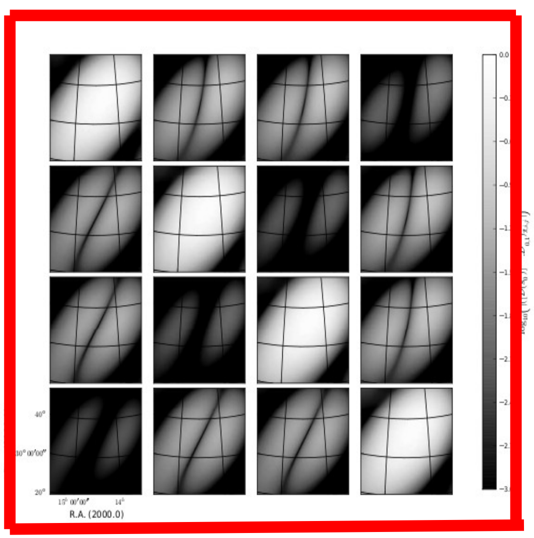
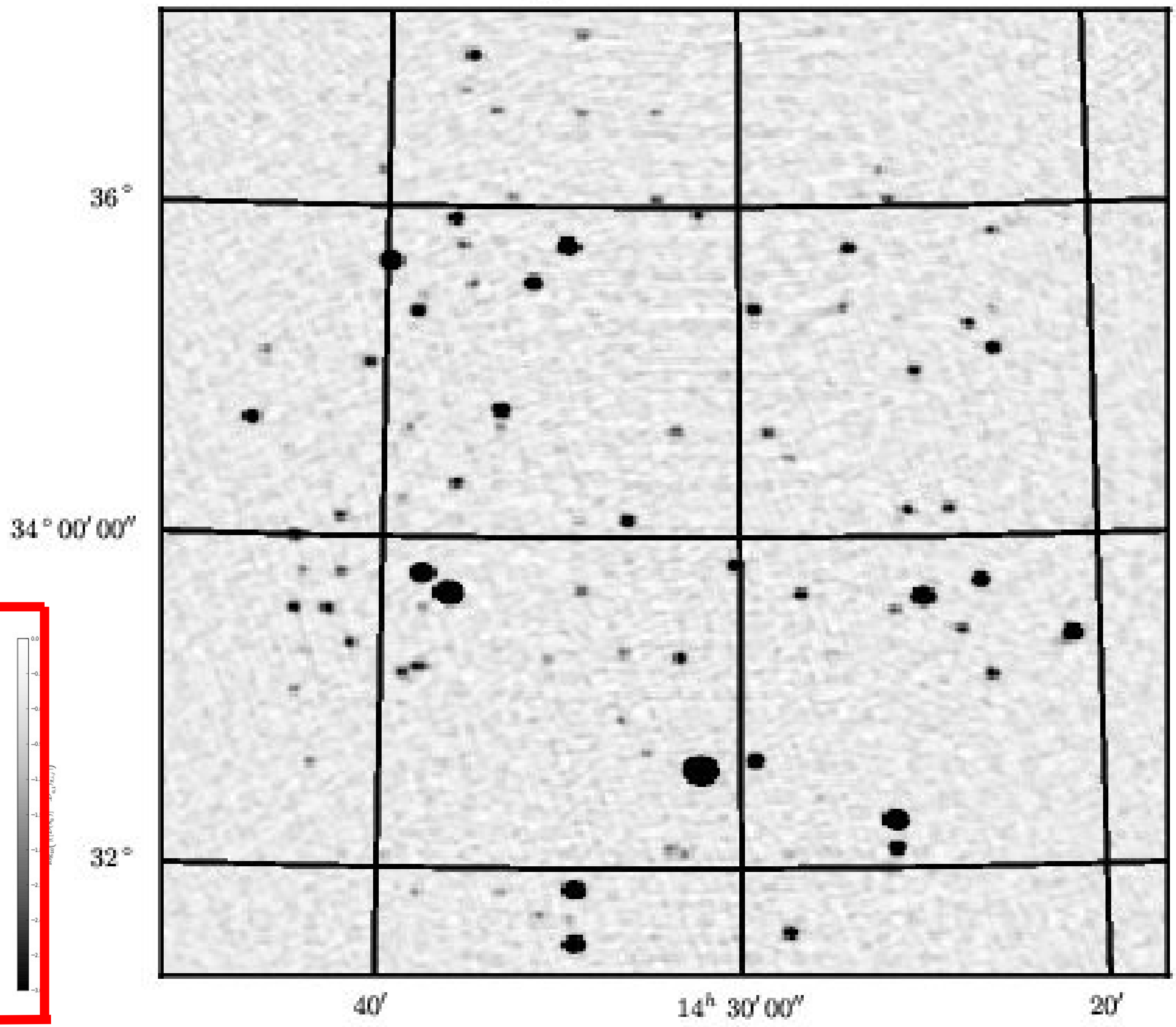
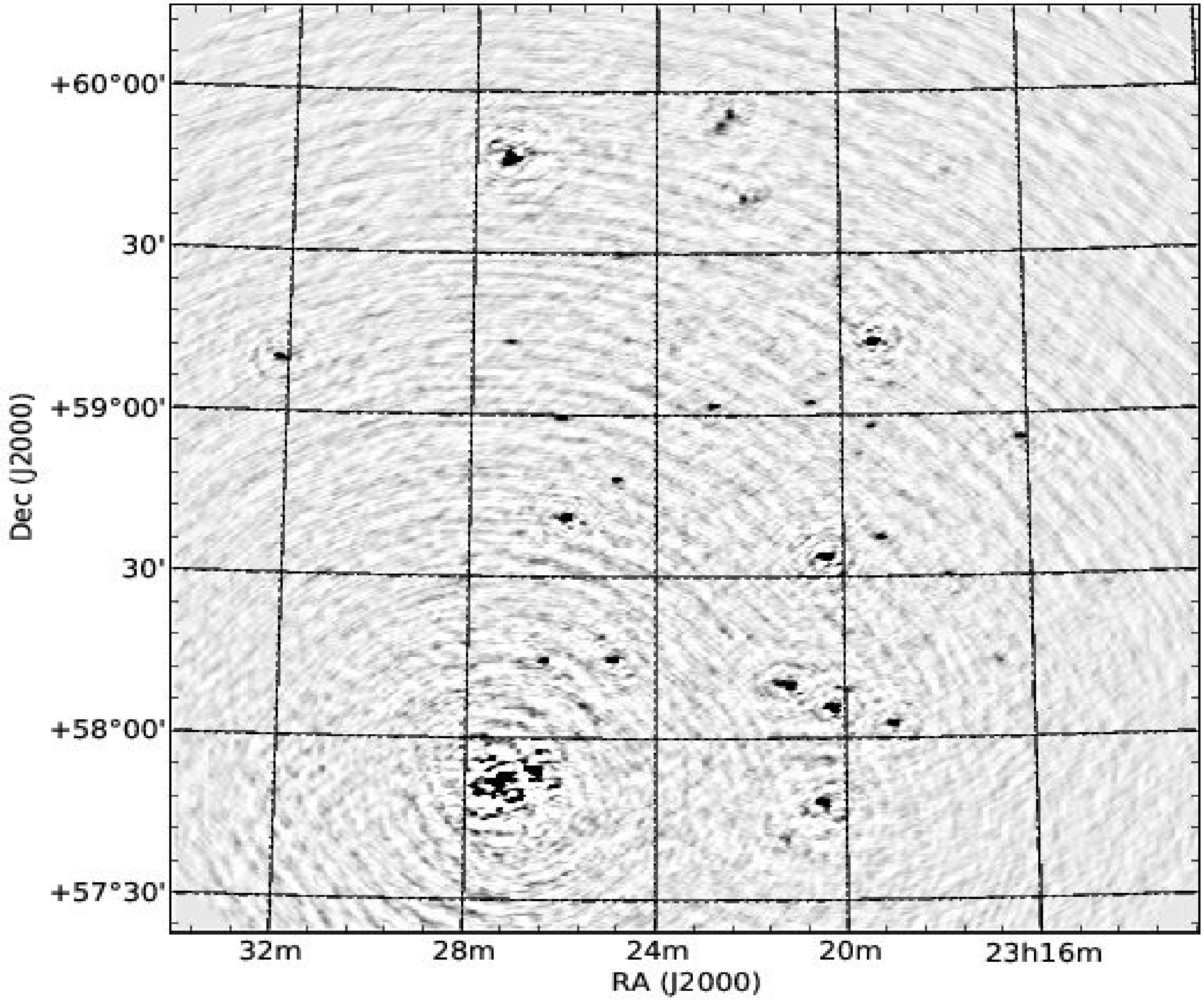


Wterm + full beam Mueller matrix



Putting ionosphere in the imager...



Simulated dataset with many point sources (LOFAR's beam+ionosphere)

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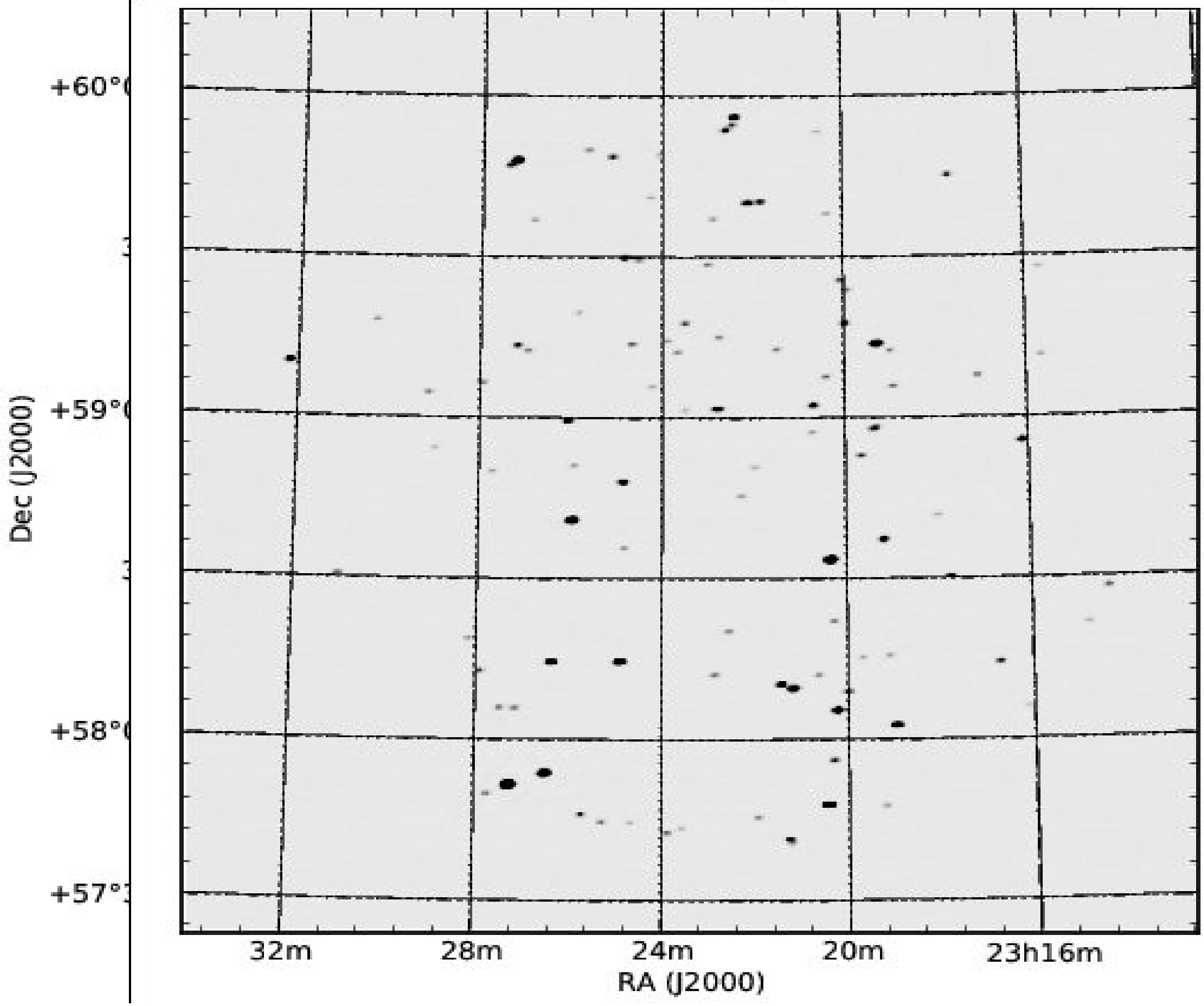

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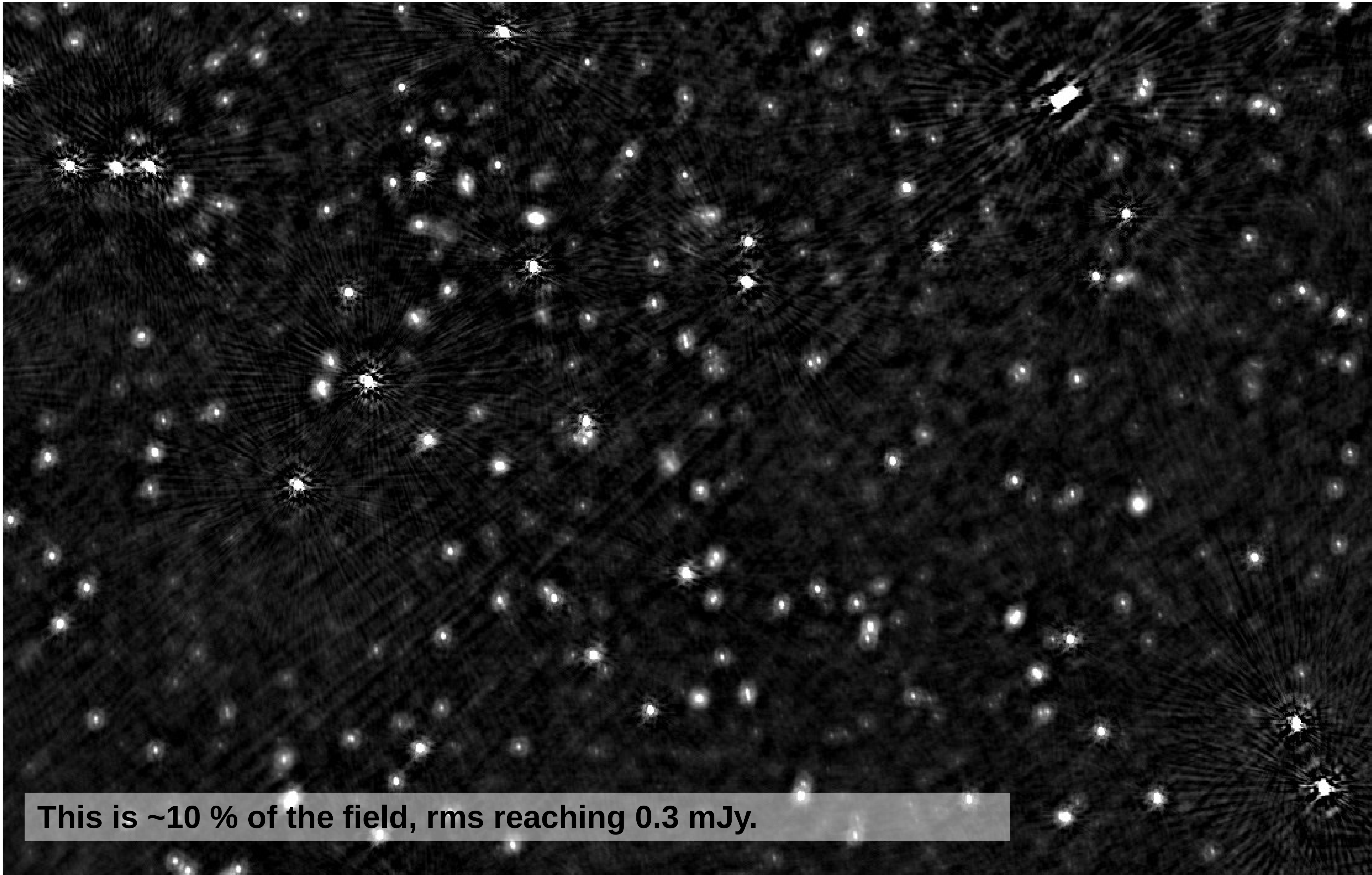
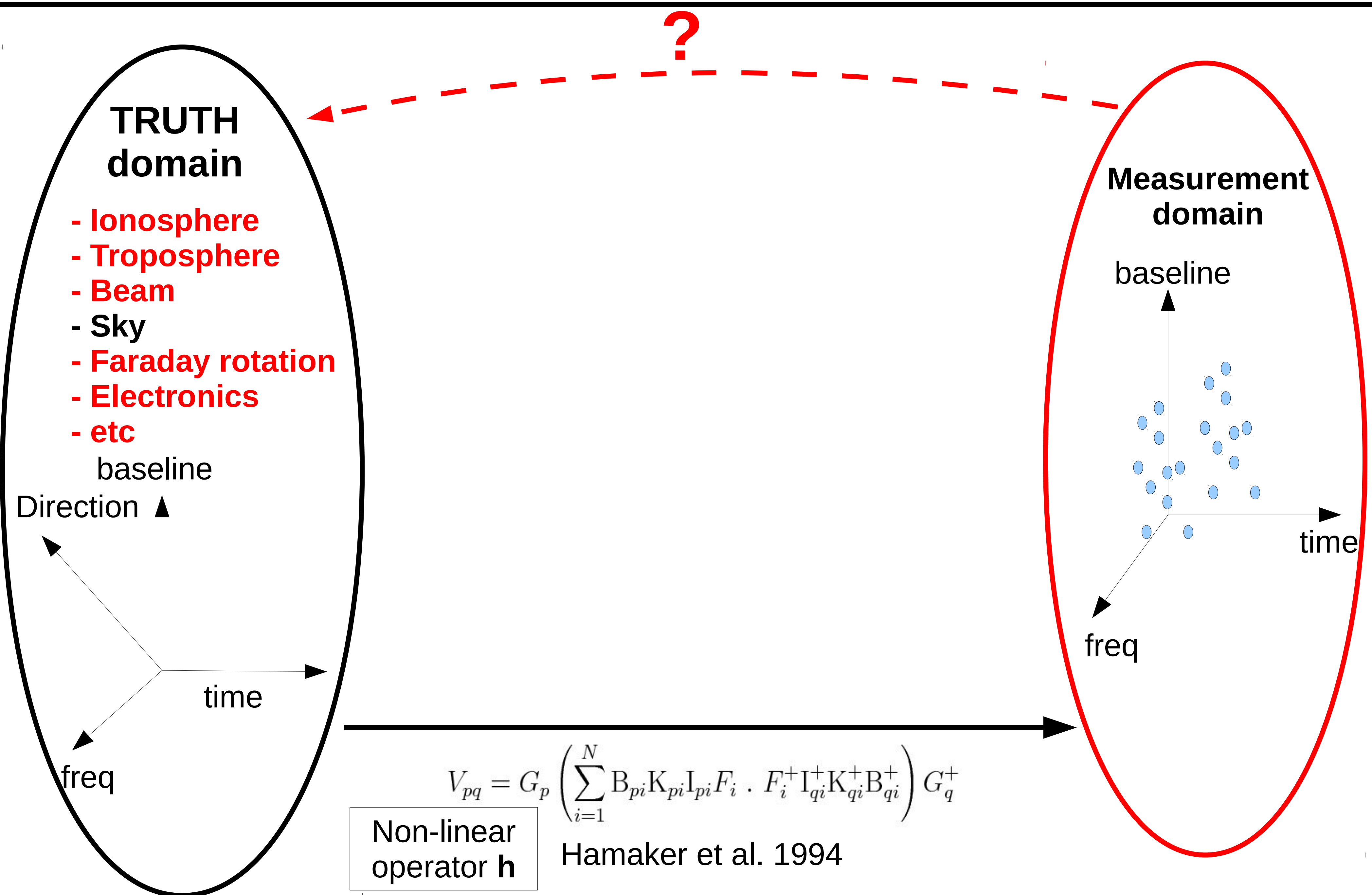


Calibration



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Hamaker et al. 1994

Calibration

TRUTH domain

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- **Troposphere**
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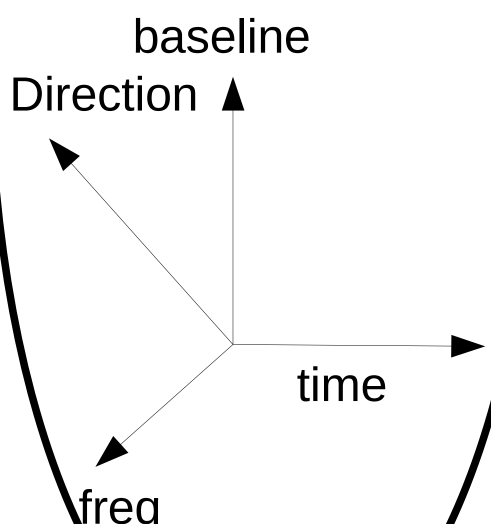
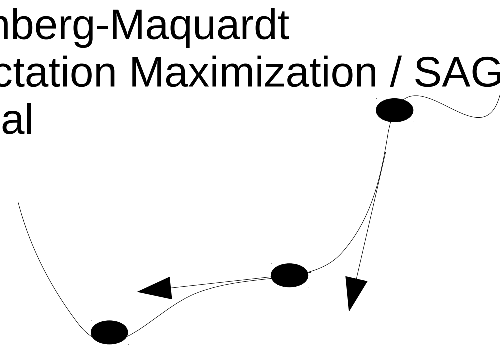
- **etc**



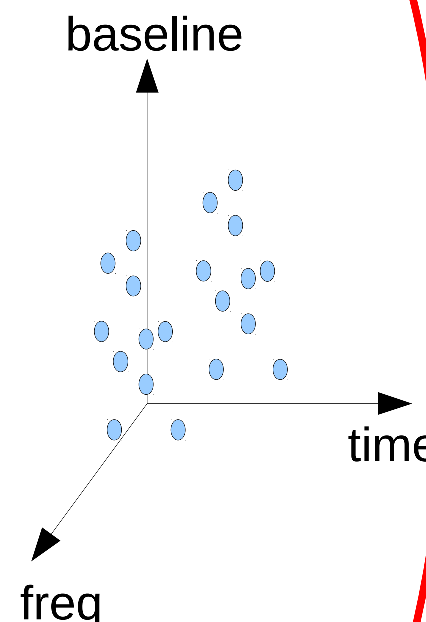
Only iterative solvers have been used so far:
- Levenberg-Maquardt
- Expectation Maximization / SAGE
- StefCal



Assuming a sky, find the maximum likelihood
- *each time/frequency bin independent*
- *each variable independent*

When solving for a large number of free parameters, give rise to ***ill-conditioning***

Measurement domain



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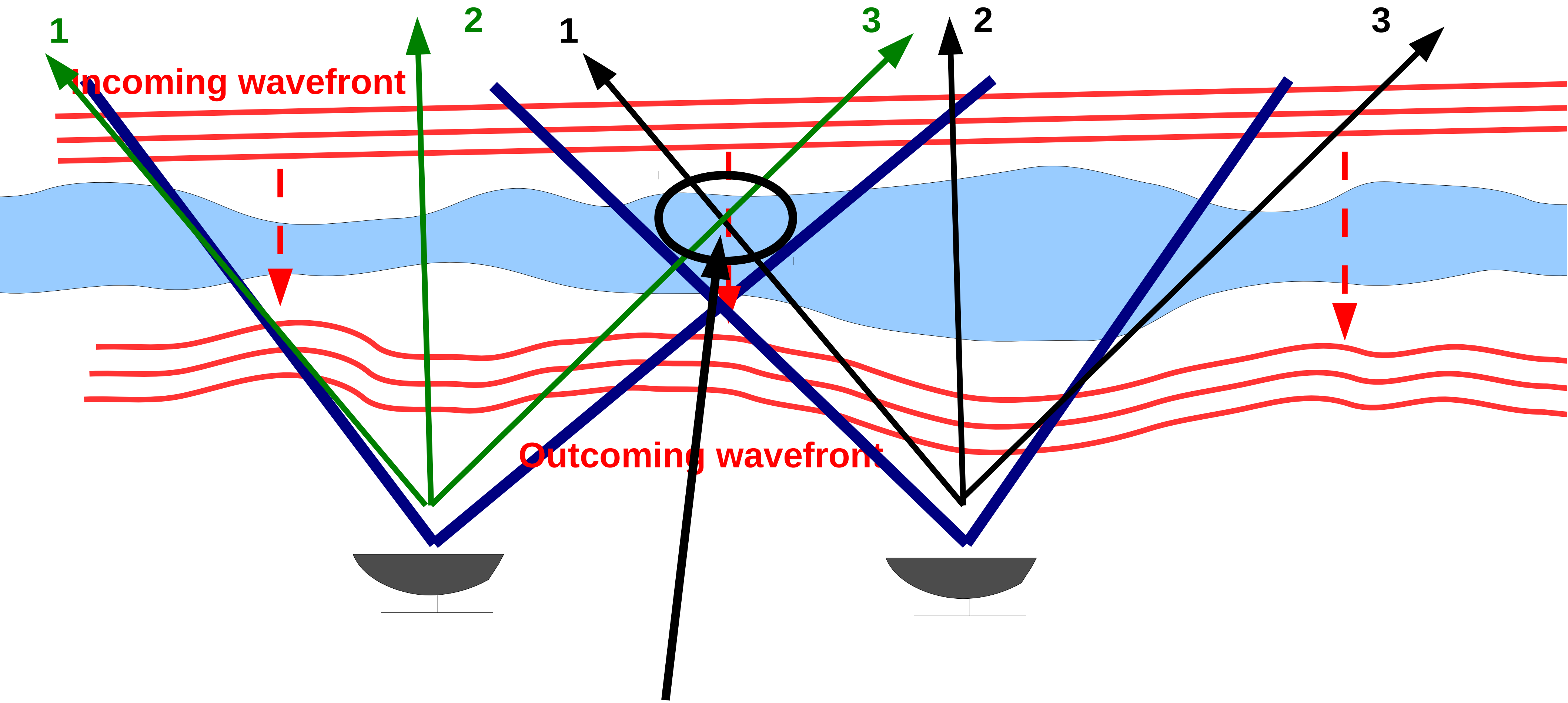
Non-linear operator **h**

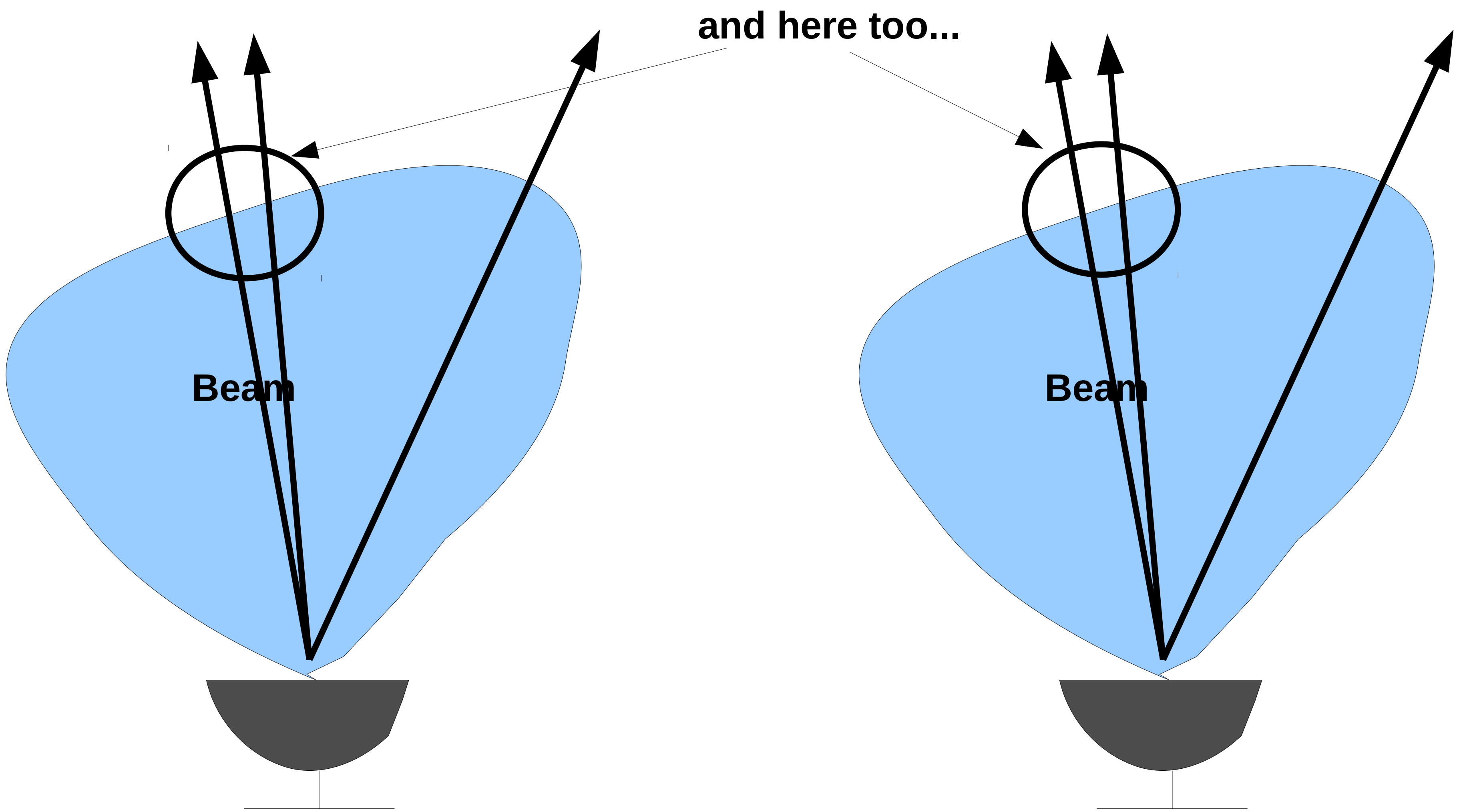
Hamaker et al. 1994

While... there is regularity in the process



Process parameters are correlated here

While... there is regularity in the process



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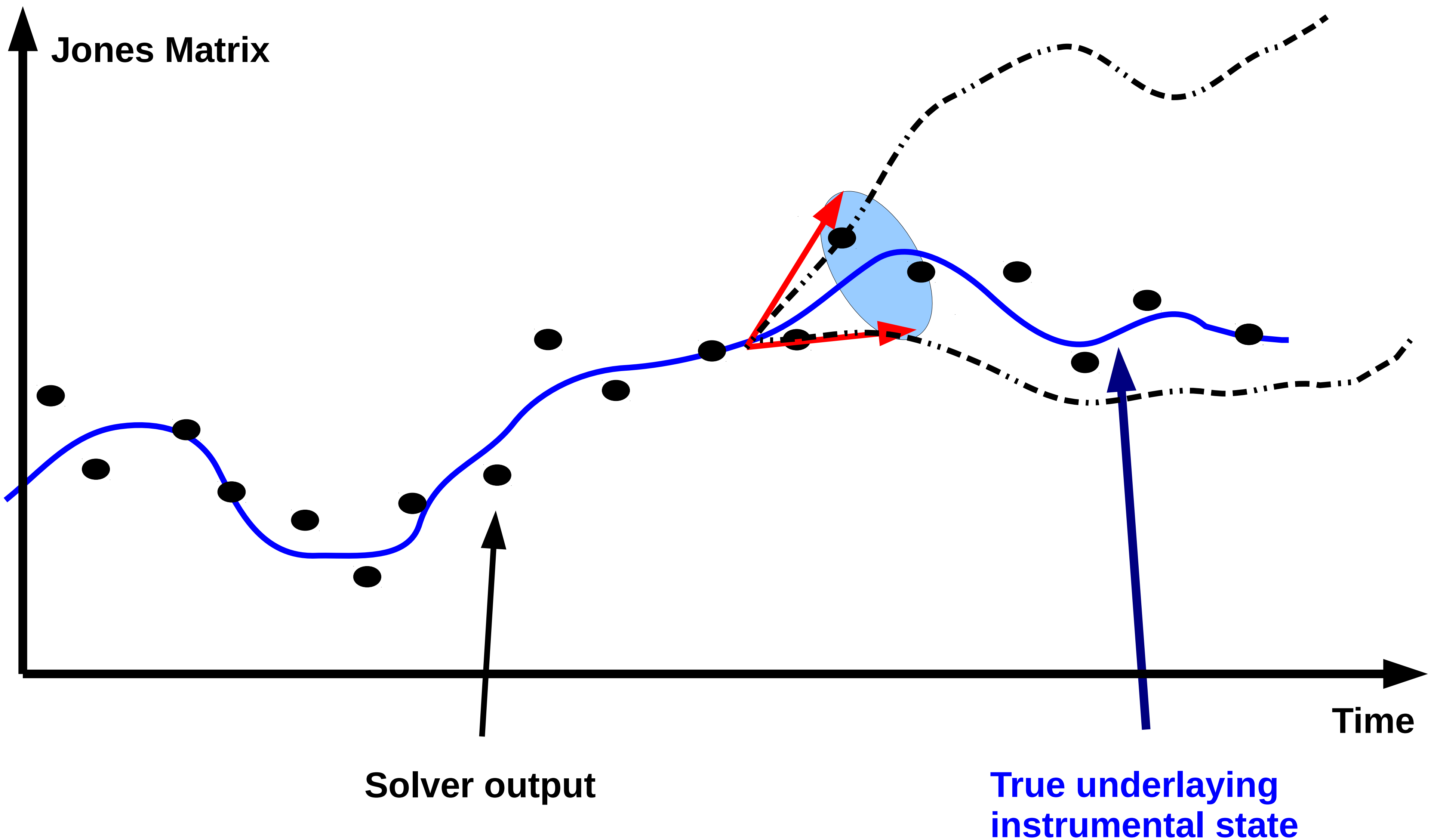

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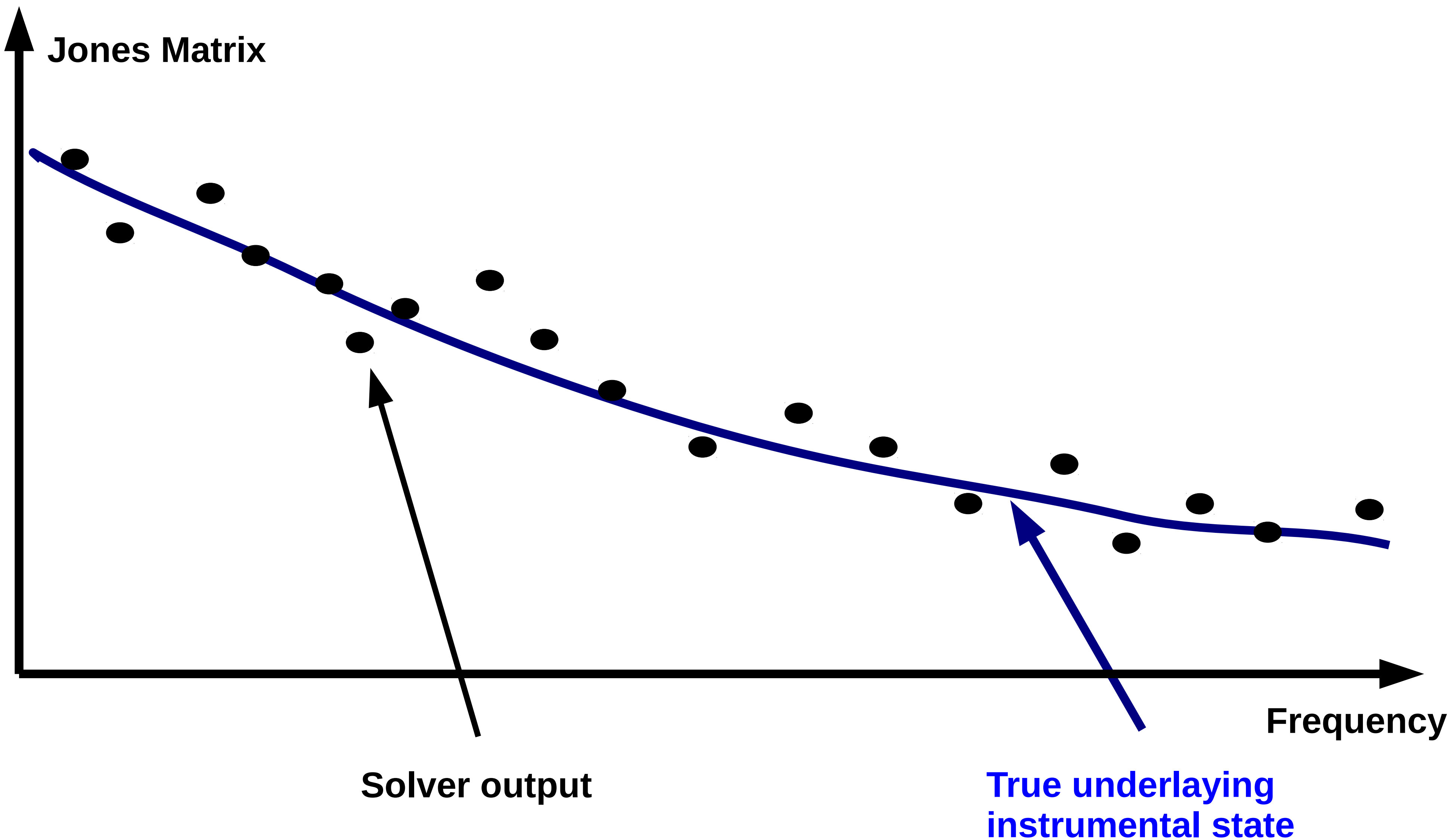
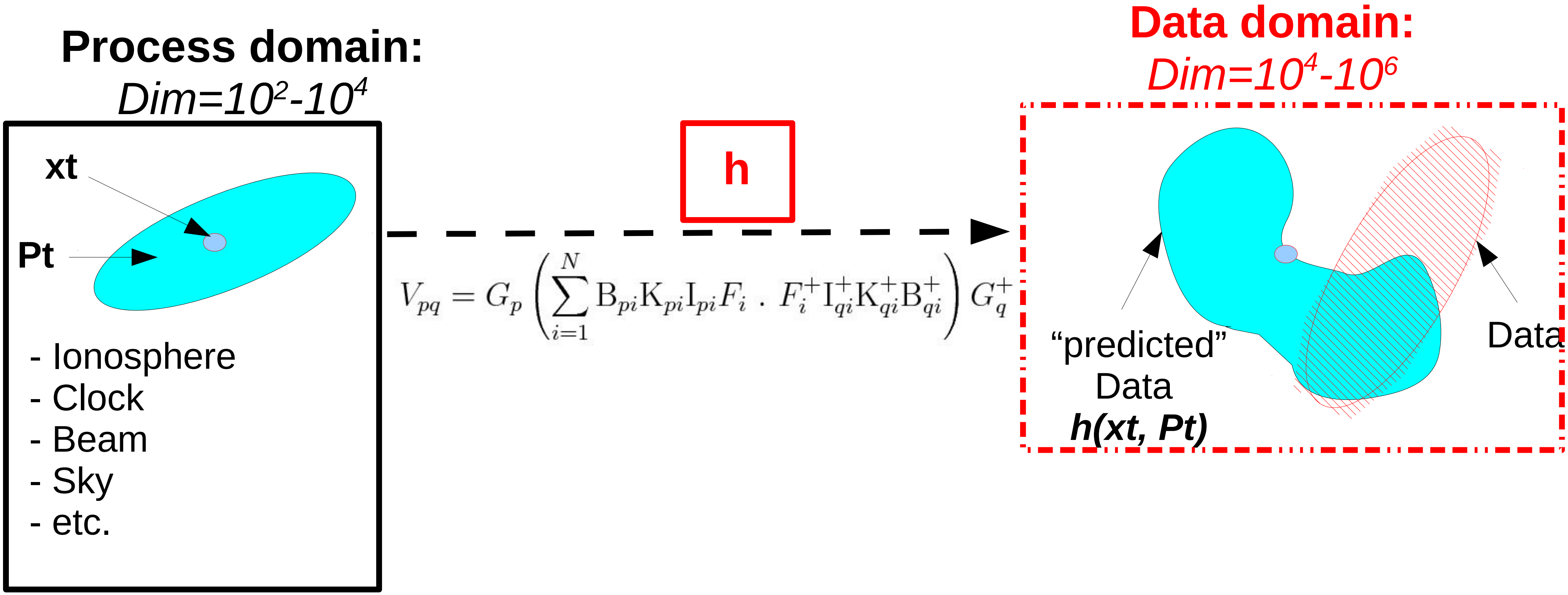


Non-linear Kalman Filters....



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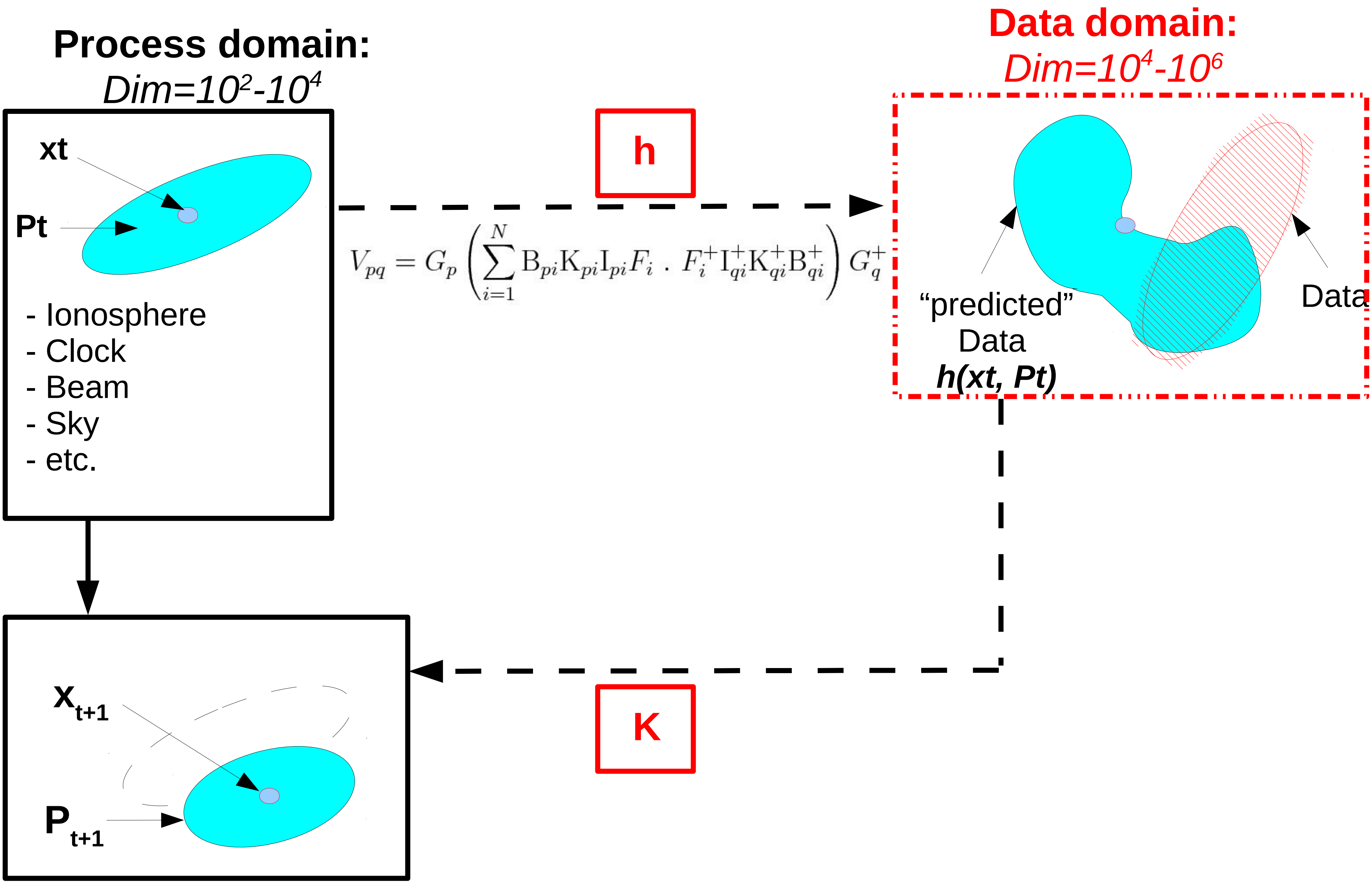

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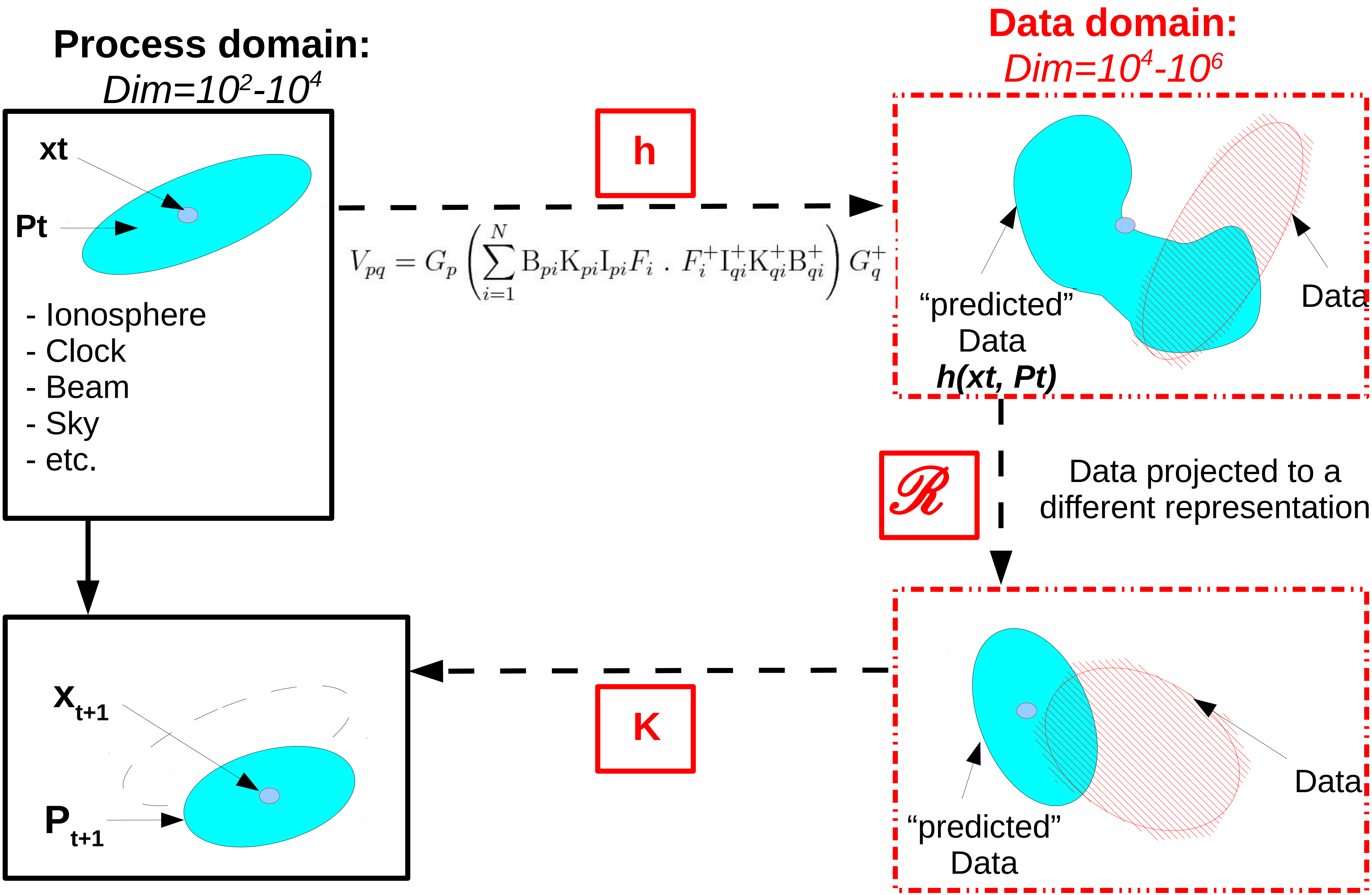
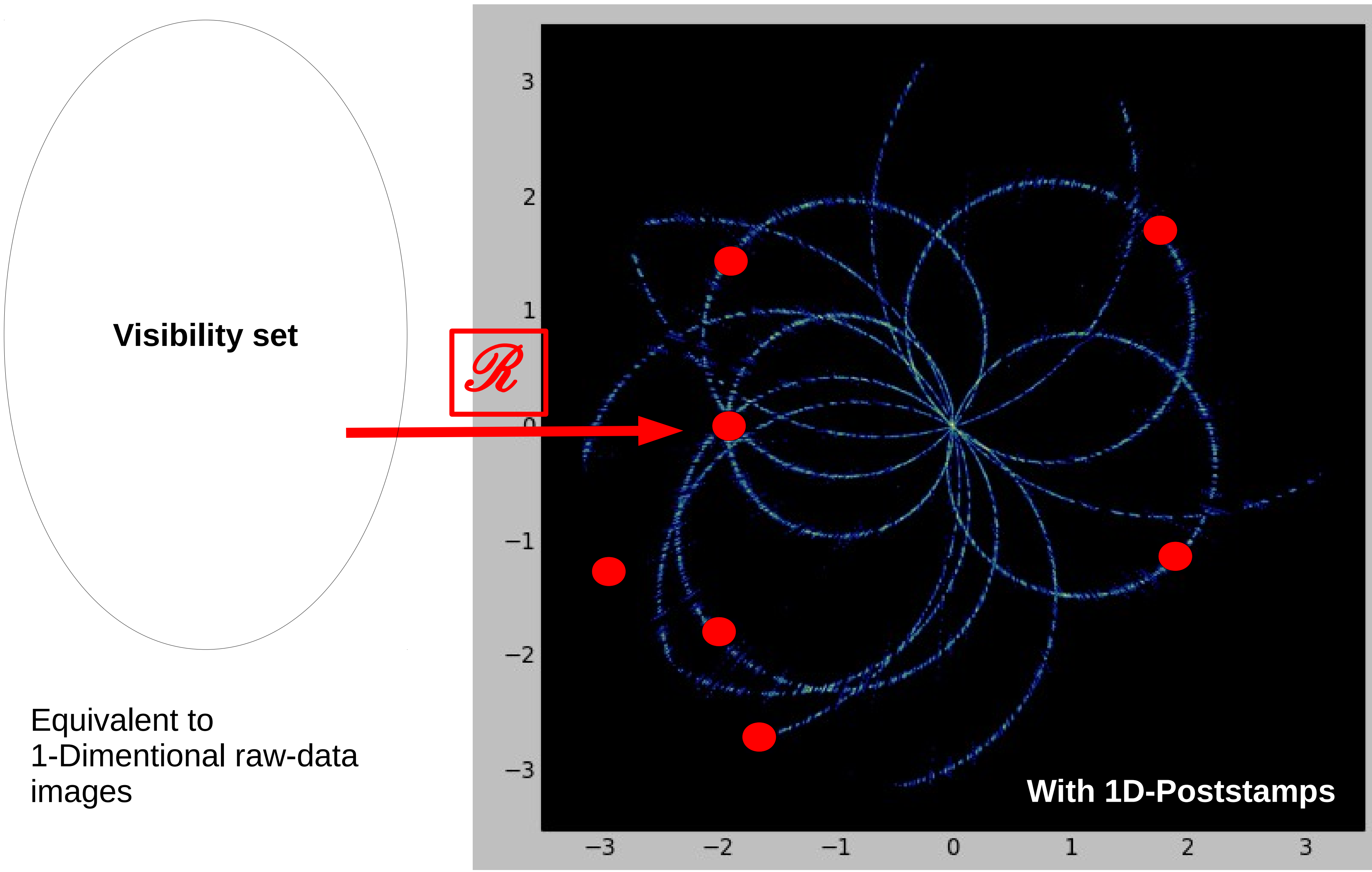


Representation issues....



Equivalent to
1-Dimentional raw-data
images

Simulated dataset (ionosphere TEC-screen + Clock drift)

Measurement Equation

$$\mathbf{V}_{(pq)t\nu} = \mathrm{h}(\mathbf{x}) = \mathbf{G}_{pt\nu}(\mathbf{x}) \left(\sum_s \mathbf{V}^s_{(pq)t\nu}(\mathbf{x}) k^s_{(pq)t\nu} \right) \mathbf{G}^H_{qt\nu}(\mathbf{x}) \tag{1}$$

$$\mathbf{V}^s_{(pq)t\nu}(\mathbf{x}) = \mathbf{D}_{pst\nu}(\mathbf{x}) \mathbf{X}_s \, \mathbf{D}^H_{qst\nu}(\mathbf{x}) \tag{2}$$

Clocks
Ionosphere

$$\mathbf{G}_{pt\nu}(\mathbf{x}) := \exp\left(2\pi i \nu \;\, \boldsymbol{\delta} \mathbf{t}_{\mathbf{p}}(\mathbf{x})\right) \mathbf{I}$$

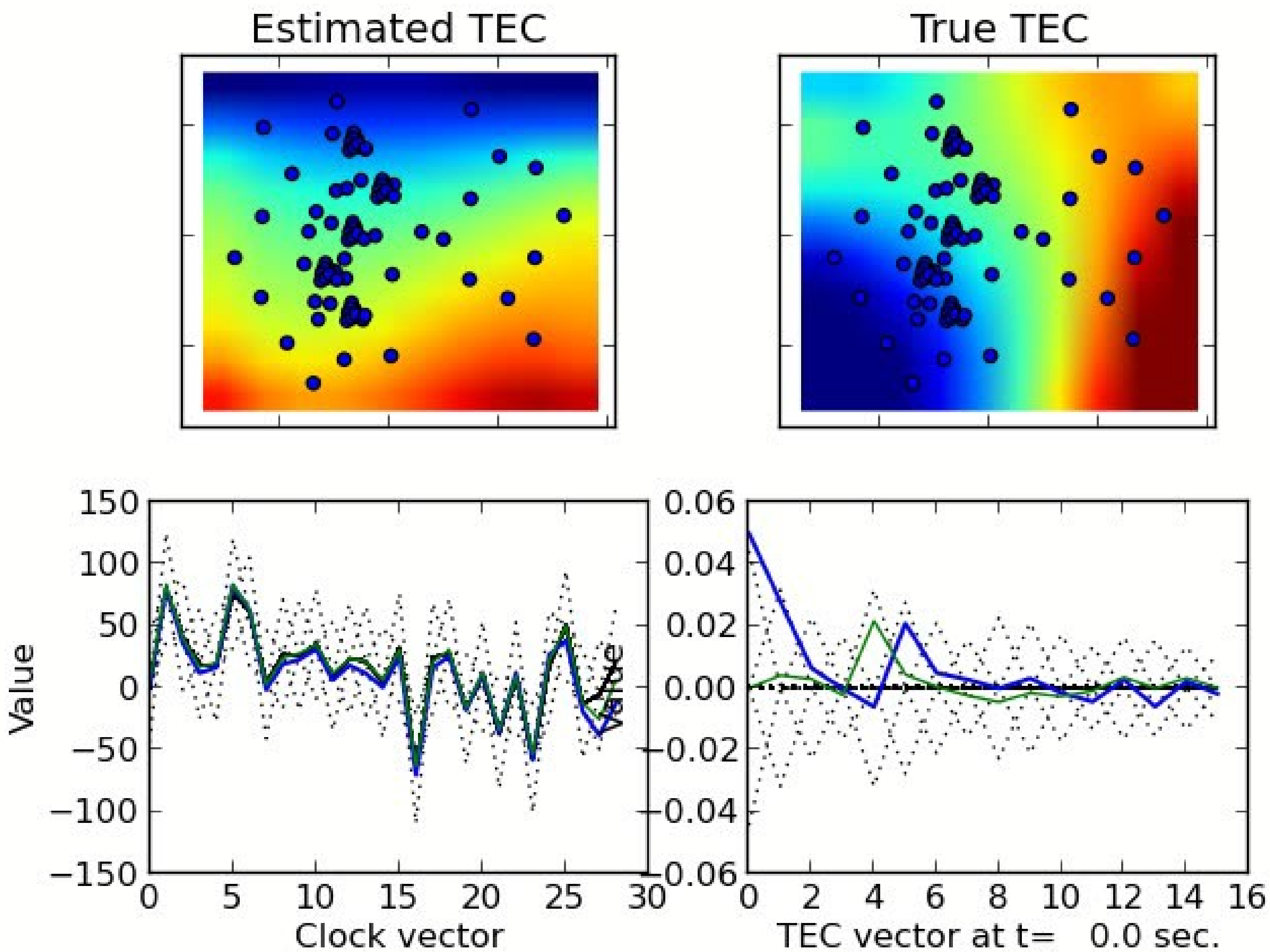
$$\mathbf{D}^d_{pt\nu}(\mathbf{x}) := \exp\left(i k \nu^{-1} \; \mathbf{T}^{\mathbf{d}}_{\mathbf{p}}(\mathbf{x}) \right) \mathbf{I}$$

- **LOFAR HBA (1500 baselines, 4 pols)**
- **30 subbands from 100 to 150 MHz**
- **S/N=5**

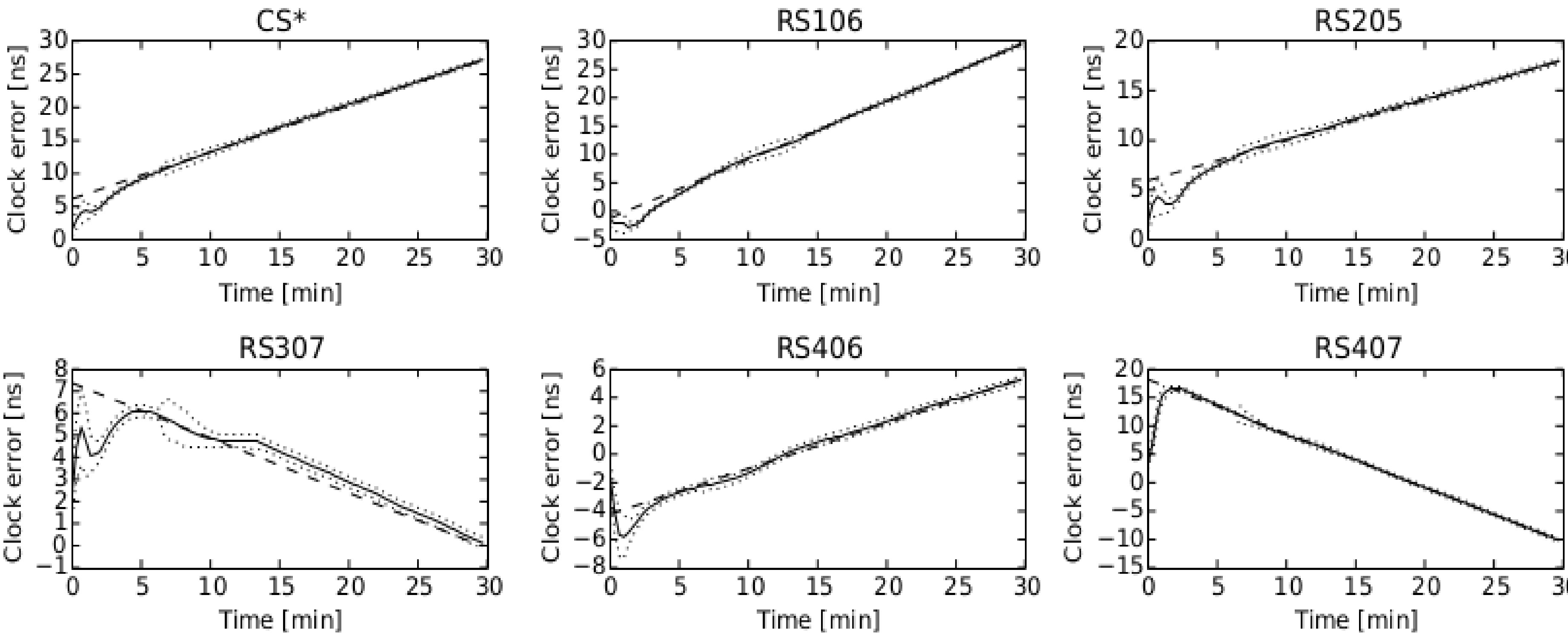
- **variable ionosphere TEC-screen**
- **Variable clock drift**

- **18.000 data points / [time bin (30s)]**

Simulated dataset (ionosphere TEC-screen + Clock drift)



Clocks solutions



Solving for the sky term ?!

- Robustness seems high enough now....
- Let's simulate a grid of sources, and solve for their position

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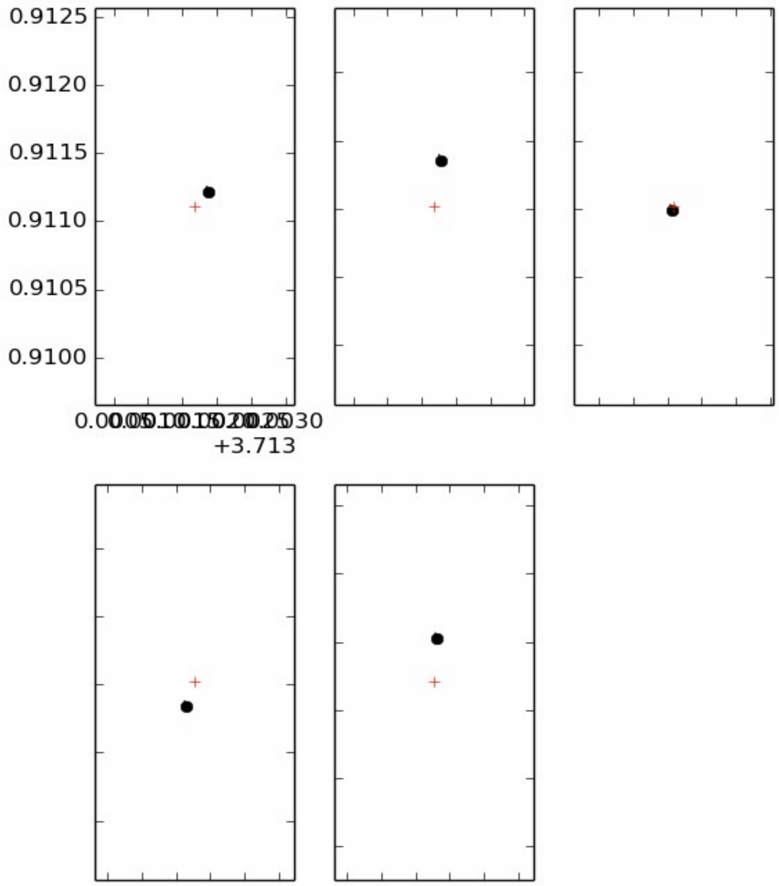
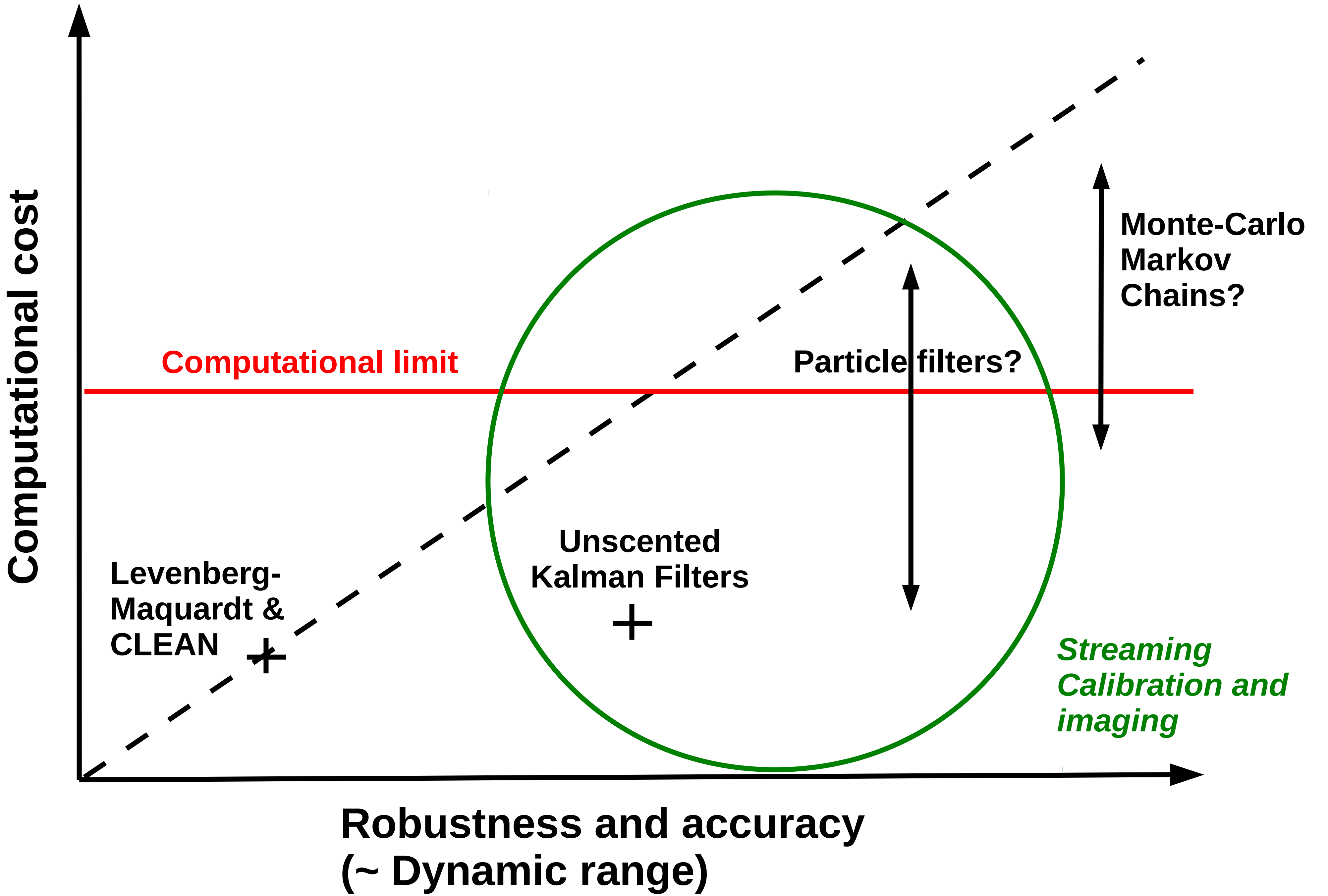


Conclusion - I



Conclusion - II

Towards a (streaming) adaptative optics
(wide field, full polarization)

- Radio interferometry is **simple to formalize** and **hard to solve**

- A LOT of things happening in the domain:
    - Imaging and deconvolution
    - New calibration techniques

- **ill conditioning** is a real issue

Filters:
- **We calibrate the raw data in the image plane**
- We **decrease** number of parameters **by orders of magnitude** (seem to properly adress ill-conditioning)
- The **solutions are physical & valid everywhere** (not only at discrete locations)
- This robustness allows to add additional degrees of freedom such as sky itself !
- Fairly fast given what it does
- Works only if instrument and sky are analytically describable (need for a “analytically stable” instrument)
- Need to access all the frequency data simultaneously (that's why it takes time to get to test the algorihm on real data)