LOFAR Sparse Image Reconstruction

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Radio-Interferometry Image Reconstruction

\[ Y = HX + N \]

Compressed Sensing Theory and Radio-Interferometry

Compressed Sensing: a sampling theorem

A non linear sampling theorem

“Signals with exactly $K$ components different from zero can be recovered perfectly from $\sim K \log N$ incoherent measurements”

Sparse recovery:  
Reconstruction via non linear processing:  
$$
\min_x ||x||_1 \quad \text{s.t.} \quad y = \Theta x
$$
Radio-Interferometry Sparse Recovery

\[ X = \Phi \alpha \]

\[
\min_{\alpha} \|\alpha\|_p^p \quad \text{subject to} \quad \|Y - H\Phi \alpha\|^2 \leq \epsilon
\]

Sparse Recovery: Example

1. **Apply mask + Noise**
2. **Sampling/Sensing**
3. **Inverse FFT**
4. **Sparse Recovery**
Compressed Sensing & LOFAR

How good is the photometry?

How well does it work on extended sources?

How good is the reconstructed image resolution?

How does CS work on LOFAR real data?
LOFAR Specific Compressed Sensing Imaging

$H_{\text{LOFAR}}$ operator much more complicated than simple FT

- Visibilities are in 3-D. Need W-Projection (see C. Tasse presentation).
- Rotation of the Earth, changing orientations $\rightarrow$ time and direction dependent effects (DDE). Need A-projection.
- Points in (U,V) space sparsely populated and non-equispaced.

Strategy:

- Use directly the $H_{\text{LOFAR}}$ implementation in the LOFAR pipeline developped by C. Tasse
- Chose wavelets (undecimated isotropic wavelets) for sparsifying the solution.
- Use minimization software developed at Saclay.
Experiment #1: Photometry

Simulated dataset

10x10 grid of point sources

Random flux densities

[1-10000] Jy

Large field of view

8°x8° centered at zenith

Widefield imaging

- CLEAN

- Sparse reconstruction

➢ recover flux densities from model images
Sparse recovery provides similar results to CLEAN.

\[\text{Experiment #1: Photometry}\]

Point source reconstruction

\[\text{Input Flux density (Jy)}\]

\[\text{Output Flux density (Jy)}\]

\[\text{Absolute Error (Jy)}\]

\[\text{Input Flux density (Jy)}\]

\(\Rightarrow\) Sparse recovery provides similar results to CLEAN.
Experiment #2: Angular separation

- Simulated LOFAR dataset
  * Core stations only (N=24)
  * ΔT=1h - ΔF=195 KHz - F=150 MHz
  * Radial cut in the Fourier (u,v) plane at R_{uv}=1.6 kλ
    ➢ restricts artificially the resolution to ~2-3 arcminutes

- Filled with simulated data
  * Two point sources of 1 Jy at zenith
  * Source angular separation = from 10” to 5’
  * Injected noise corresponding to SNR = 2.7, 8.9, 16 and 2000 (noiseless)

- Imaging with CLEAN and Sparse recovery
Experiment #2: Angular separation

CLEAN

Sparse reconstruction
Sparse Recovery

- Sparse Recovery resolution improved by at least 2 compared the CLEAN beam.
- Recovered « sub-beam » sources have correct fluxes (~2% error) & positions
Experiment #2: Angular separation

- On noisy data, (rough) measurement of the source separability angle.

Rayleigh criterion

Separated sources when decrease > 23%

Effective source separability vs. SNR

Angular separation (°) vs. SNR

$\Rightarrow$ Sparse reconstruction: angular separation improved by 2 for $\text{SNR} > 10$, and converges to CLEAN resolution at low SNR regimes.
**Experiment #3: Extended source**

- VLA 21-cm image of W50 + empty simulated LOFAR dataset
- Set to an arbitrary flux scale and converted to visibilities (AWimager)

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**Model image**

![Model image](image1)

**Dirty image**

![Dirty image](image2)

**FFT + (u,v) Sampling**

![FFT + (u,v) Sampling](image3)

**VLA @ 21 cm**

![VLA @ 21 cm](image4)

**(u,v) coverage**

![Coverage](image5)
Experiment #3: Extended source

- Using CLEAN, Multiscale CLEAN and Sparse reconstruction

**RMS error =**

- **CLEAN:** 3.50
- **Multiscale CLEAN:** 3.28
- **Sparse Reconstruction:** 0.76
Experiment #3: Extended source

- Using CLEAN, Multiscale CLEAN and Sparse reconstruction

CLEAN

Multiscale CLEAN

Sparse Reconstruction

RMS error = 3.50

RMS error = 3.28

RMS error = 0.76
Experiment #4: Real data

Cygnus A

F = 151 MHz - ΔF = 195 kHz
ΔT = 6 Hr
36 LOFAR Stations

(dataset courtesy of John Mckean)

CLEAN

- Pixel = 1''    size = 512 x 512
- Threshold = 0.5 mJy
- Weighting = super uniform

Restored image

Total Flux density = 9393 Jy

Residua

Residual std-dev = 2.65 Jy/beam
Right Ascension
Declination

Multi-Scale CLEAN

F = 151 MHz - ΔF = 195 kHz
ΔT = 6 Hr
36 LOFAR Stations
(dataset courtesy of John Mckean)

Multi-Scale CLEAN

- Pixel = 1'' size = 512 x 512
- Threshold = 0.5 mJy
- Weighting = super uniform
- Scales = [0, 5, 10, 15, 20] pixels

Restored image
Total Flux density = 10553 Jy

Residuals
Residual std-dev = 0.26 Jy/beam
**Cygnus A**

Sparse Reconstruction

- **Right Ascension**
- **Declination**

- **F = 151 MHz - ΔF = 195 kHz**
  - ΔT = 6 Hr
  - 36 LOFAR Stations

*(dataset courtesy of John Mckean)*

### Sparse Reconstruction

- **Pixel = 1''** size = $512 \times 512$
- **Threshold = 0.5 mJy**
- **Weighting = super uniform**
- **Scales = 7 wavelets scales**
- **Minimization algorithm: FISTA**
  - Fast Iterative Shrinkage-Thresholding Algorithm

**Restored image**

Total Flux density = 10506 Jy

**Residuals**

Residual std-dev = 0.05 Jy/beam
Conclusions

✓ Sparse recovery is a totally new imaging method for LOFAR and other modern interferometers.

✓ Experimental results are good
  ▪ Photometry: similar to CLEAN on point sources.
  ▪ Resolution: improved by a factor 2 for SNR > 10.
  ▪ Extended objects reconstruction much better than CLEAN and Multiscale CLEAN.
  ▪ Improved image quality (RMS better by factor 5 compared to CLEAN)

✓ Will continue to develop (CLEAN has had 40 years)


✓ Papers
  ▪ Journal Paper in prep.