# LOFAR Sparse Image Reconstruction


## Jean-Luc Starck

## CEA Saclay

www.cosmostat.org


H. Garsden

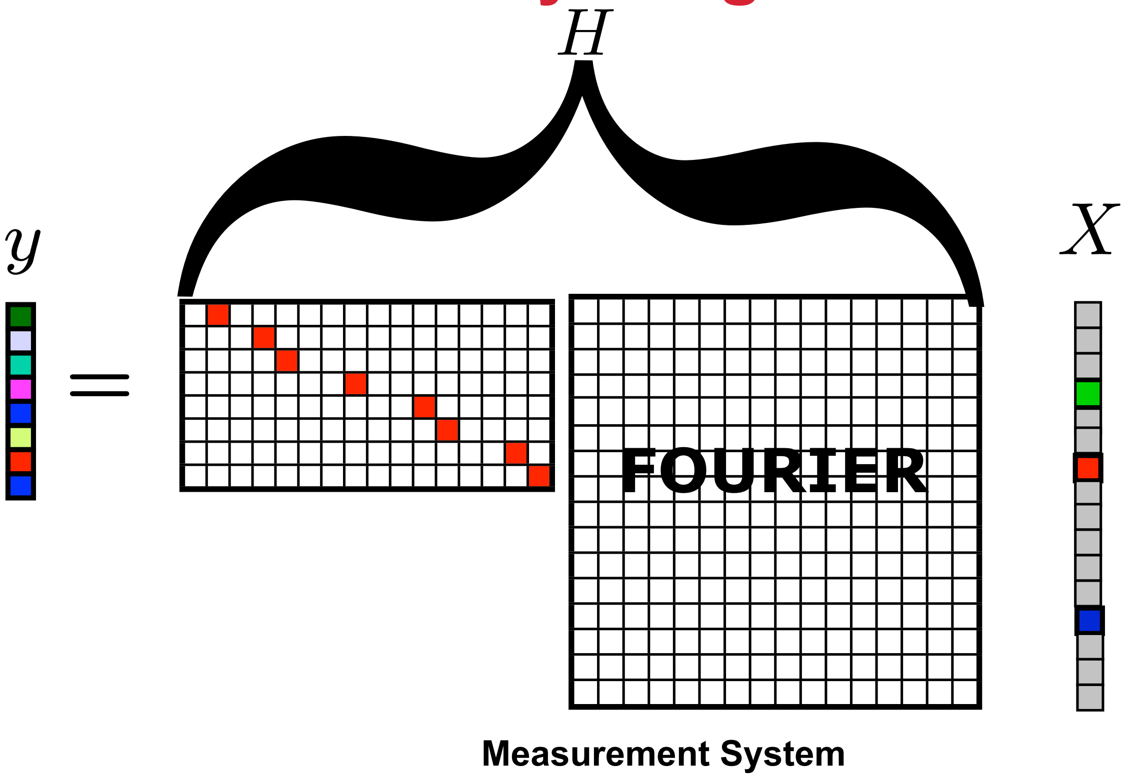
J. Girard

S. Corbel

C. Tasse

A. Woiselle

# Radio-Interferometry Image Reconstruction




$$Y = HX + N$$

**Compressed Sensing Theory and Radio-Interferometry**

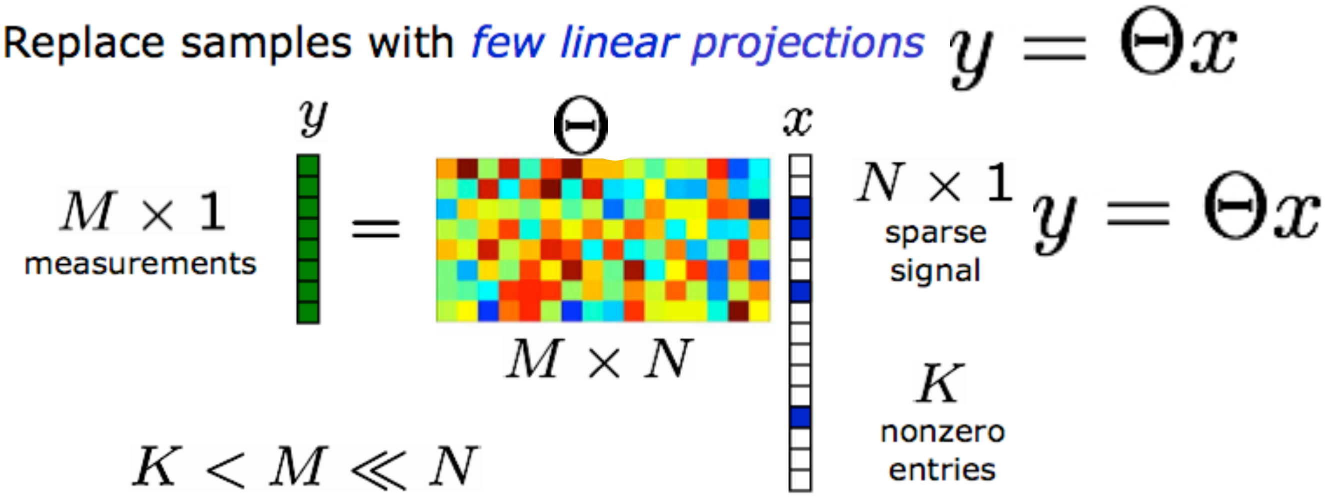
==> See (McEwen et al, 2011; Wenger et al, 2010; Wiaux et al, 2009; Cornwell et al, 2009; Suskimo, 2009; Feng et al, 2011; Garsden, Starck and Corbel, 2013).

# Compressed Sensing: a sampling theorem


* E. Candès and T. Tao, "Near Optimal Signal Recovery From Random Projections: Universal Encoding Strategies? ", IEEE Trans. on Information Theory, 52, pp 5406-5425, 2006.
* D. Donoho, "Compressed Sensing", IEEE Trans. on Information Theory, 52(4), pp. 1289-1306, April 2006.
* E. Candès, J. Romberg and T. Tao, "Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information", IEEE Trans. on Information Theory, 52(2) pp. 489 - 509, Feb. 2006.


<u>A non linear sampling theorem</u>

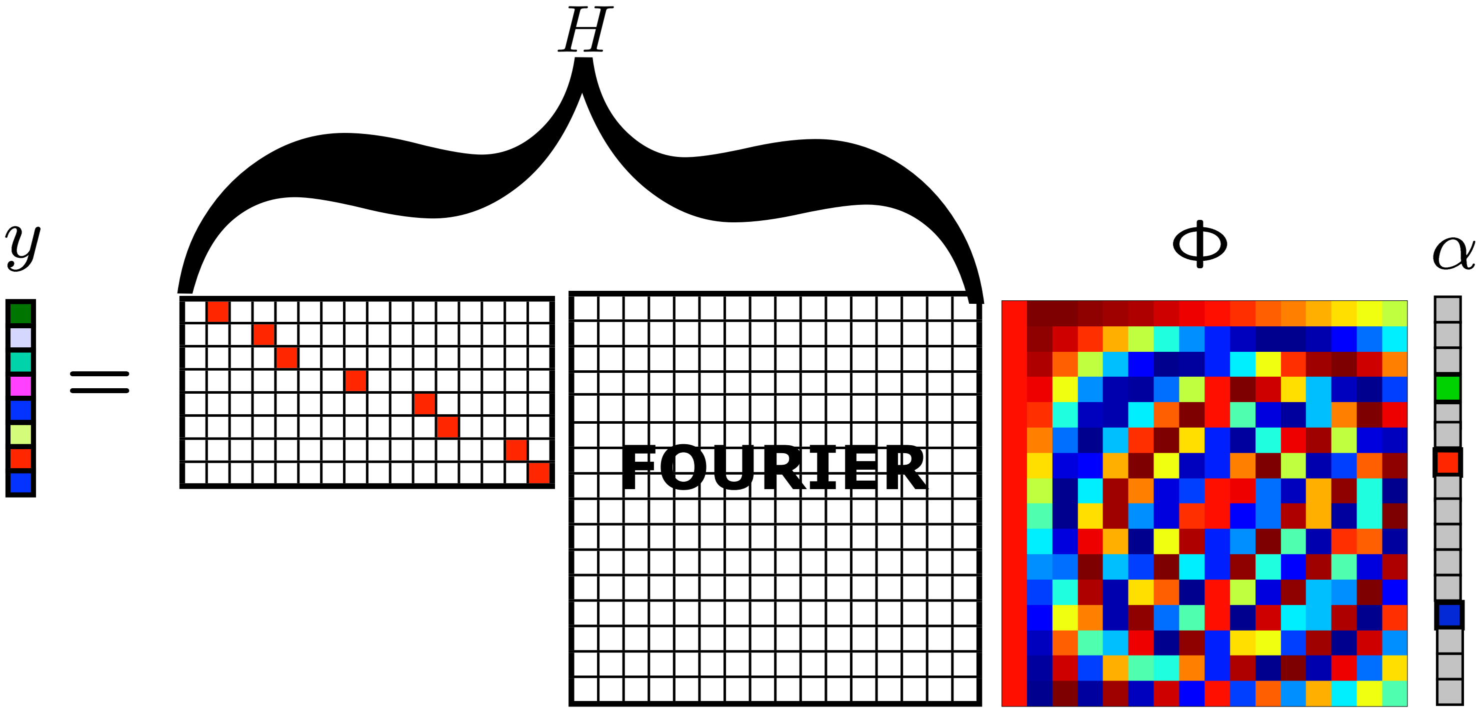
**"Signals with exactly K components different from zero can be recovered perfectly from ~ K log N incoherent measurements"**




**Sparse recovery:**
Reconstruction via non linear processing:

$$\min_x \|x\|_1 \quad \text{s.t.} \quad y = \Theta x$$

# Radio-Interferometry Sparse Recovery



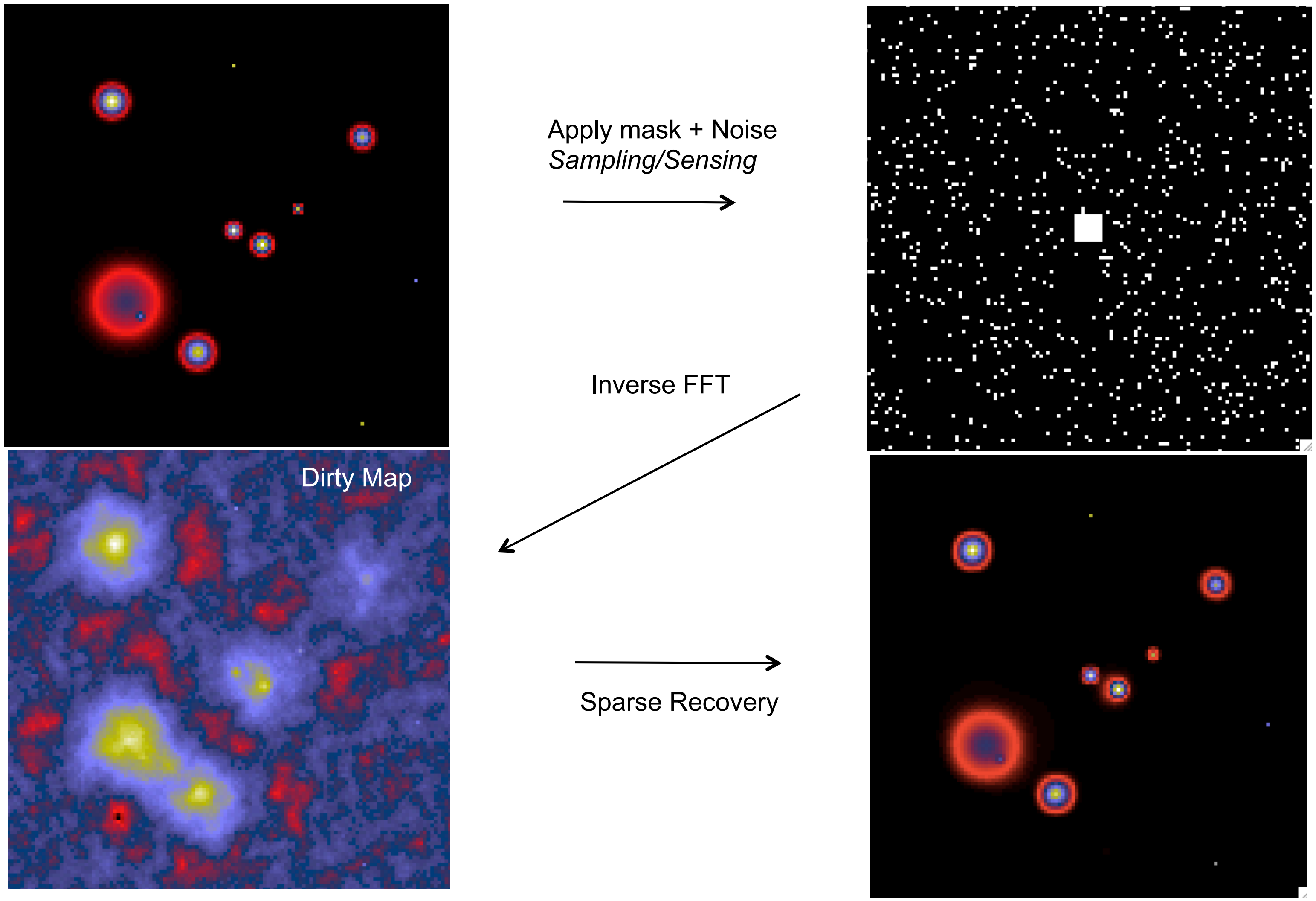

Measurement System

$$X = \Phi\alpha$$

$$\min_{\alpha} \|\alpha\|_p^p \quad \text{subject to} \quad \|Y - H\Phi\alpha\|^2 \leq \epsilon$$

Refs: Vonesch et al, 2007; Elad et al 2008; Wright et al., 2008; Nesterov, 2008 and Beck-Teboulle, 2009; Blumensath, 2008; Maleki et Donoho, 2009, Starck et al, 2010, Raguet, Fadili, and Peyre, 2012; Vu , 2013 ; etc.

# Sparse Recovery: Example



# Compressed Sensing & LOFAR

How good is the photometry ?

How well does it work on extended sources ?

How good is the reconstructed image resolution ?

How does CS work on LOFAR real data ?

# LOFAR Specific Compressed Sensing Imaging

$H_{LOFAR}$ operator much more complicated than simple FT

- Visibilities are in 3-D. Need W-Projection (see C. Tasse presentation).
- Rotation of the Earth, changing orientations -> time and direction dependent effects (DDE). Need A-projection.
- Points in (U,V) space sparsely populated and non-equispaced.

## Strategy:

- Use directly the $H_{LOFAR}$ implementation in the LOFAR pipeline developped by C. Tasse

- Chose wavelets (undecimated isotropic wavelets) for sparsifying the solution.

- Use minimization software developed at Saclay.

# Experiment #1: Photometry

**Simulated dataset**

10x10 grid of
point sources

**Random flux densities**

[1-10000] Jy

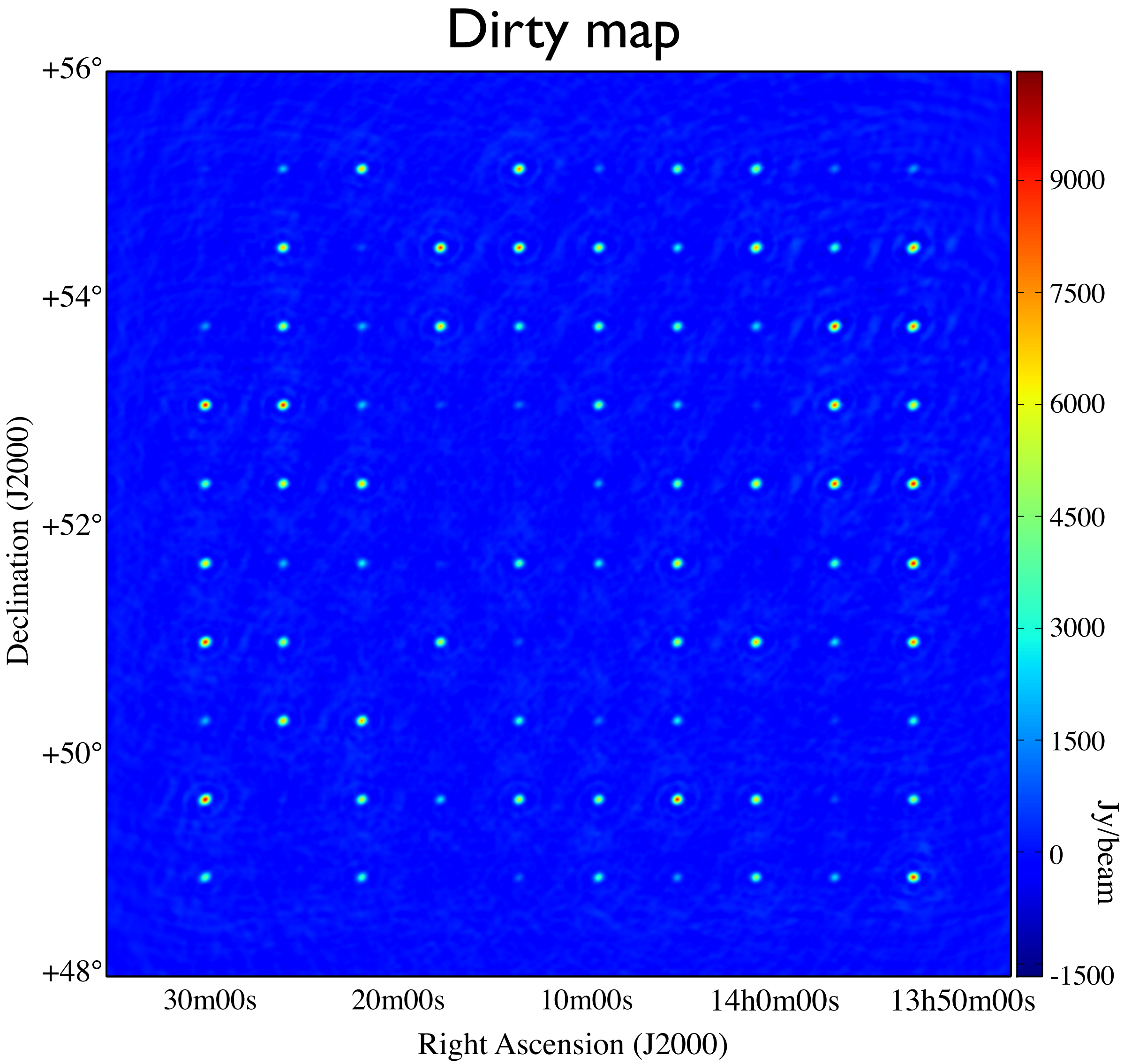
**Large field of view**

8°x8°
centered at zenith

**Widefield imaging**

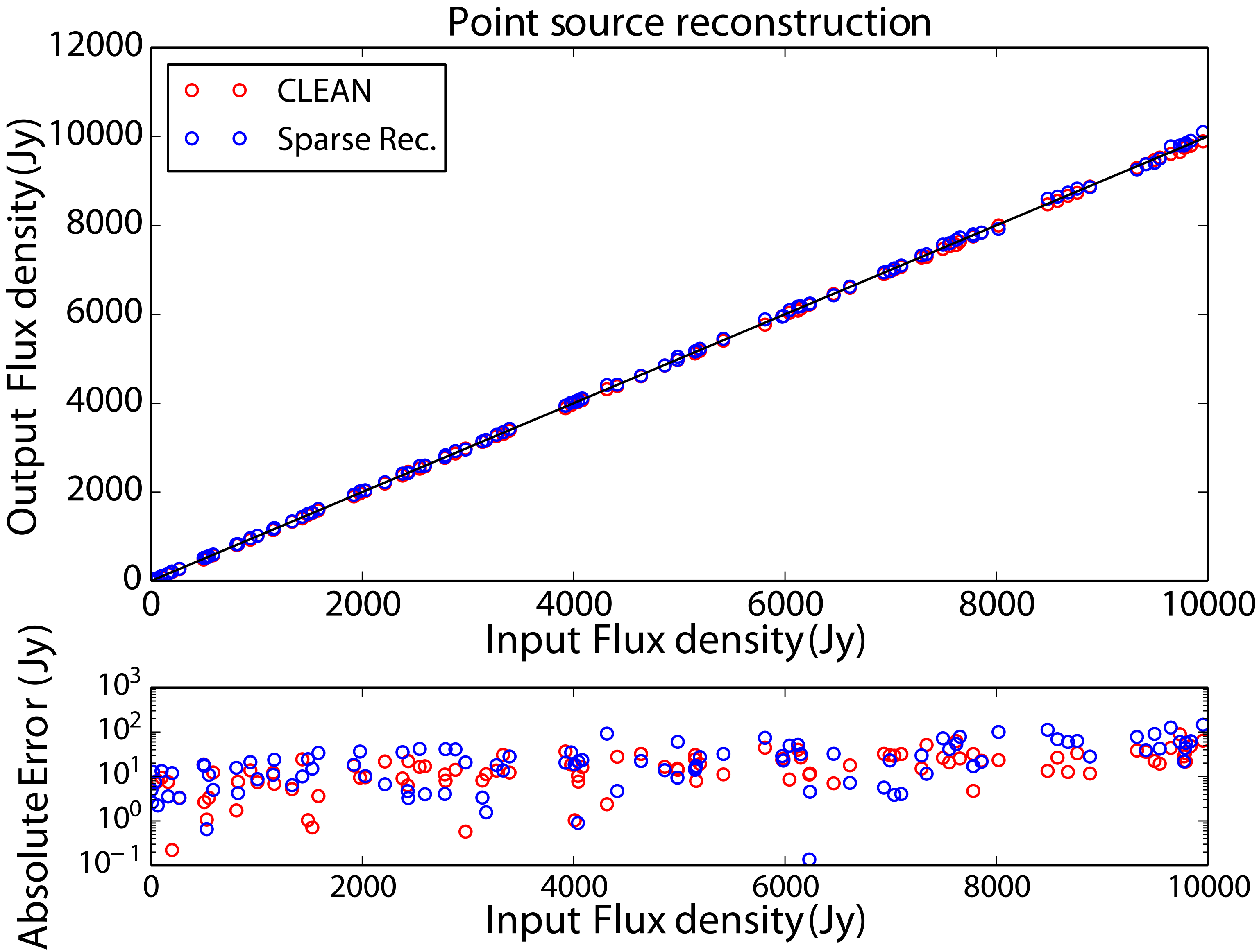
- CLEAN

- Sparse reconstruction




➢ recover flux densities from model images

# Experiment #1: Photometry




==> Sparse recovery provides **similar** results to CLEAN

# Experiment #2: Angular separation

## - Simulated LOFAR dataset

* Core stations only (N=24)

* $\Delta T$=1h - $\Delta F$=195 KHz - F=150 MHz

* Radial cut in the Fourier (u,v) plane at $R_{uv}$=1.6 k$\lambda$

➢ restricts artificially the resolution to ~2-3 arcminutes

## - Filled with simulated data

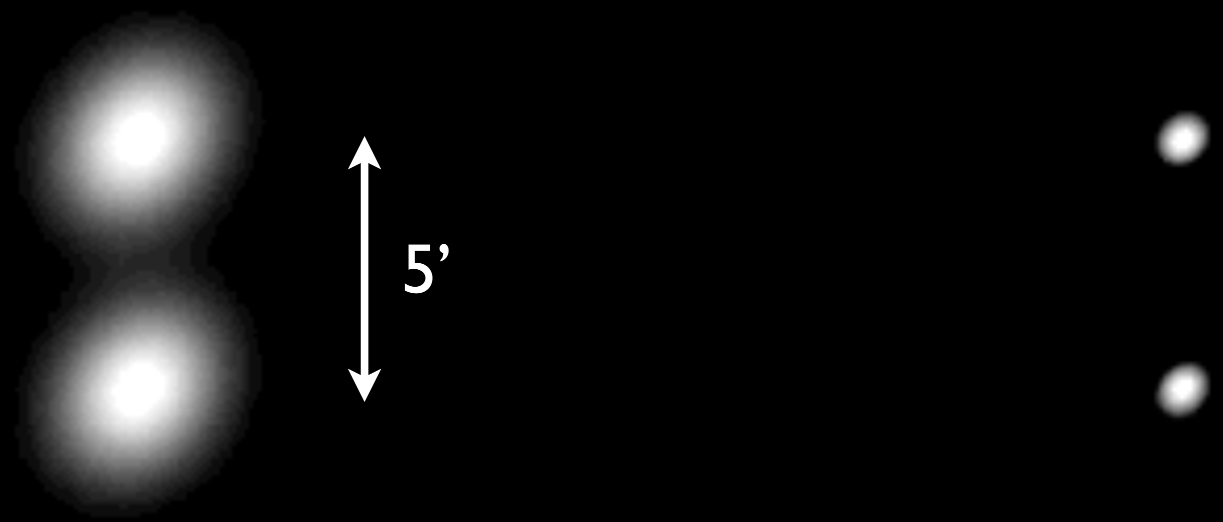
* Two point sources of 1 Jy at zenith

* Source angular separation = from 10” to 5’

* Injected noise corresponding to SNR = 2.7, 8.9, 16 and 2000 (noiseless)

## - Imaging with CLEAN and Sparse recovery

# Experiment #2: Angular separation




CLEAN

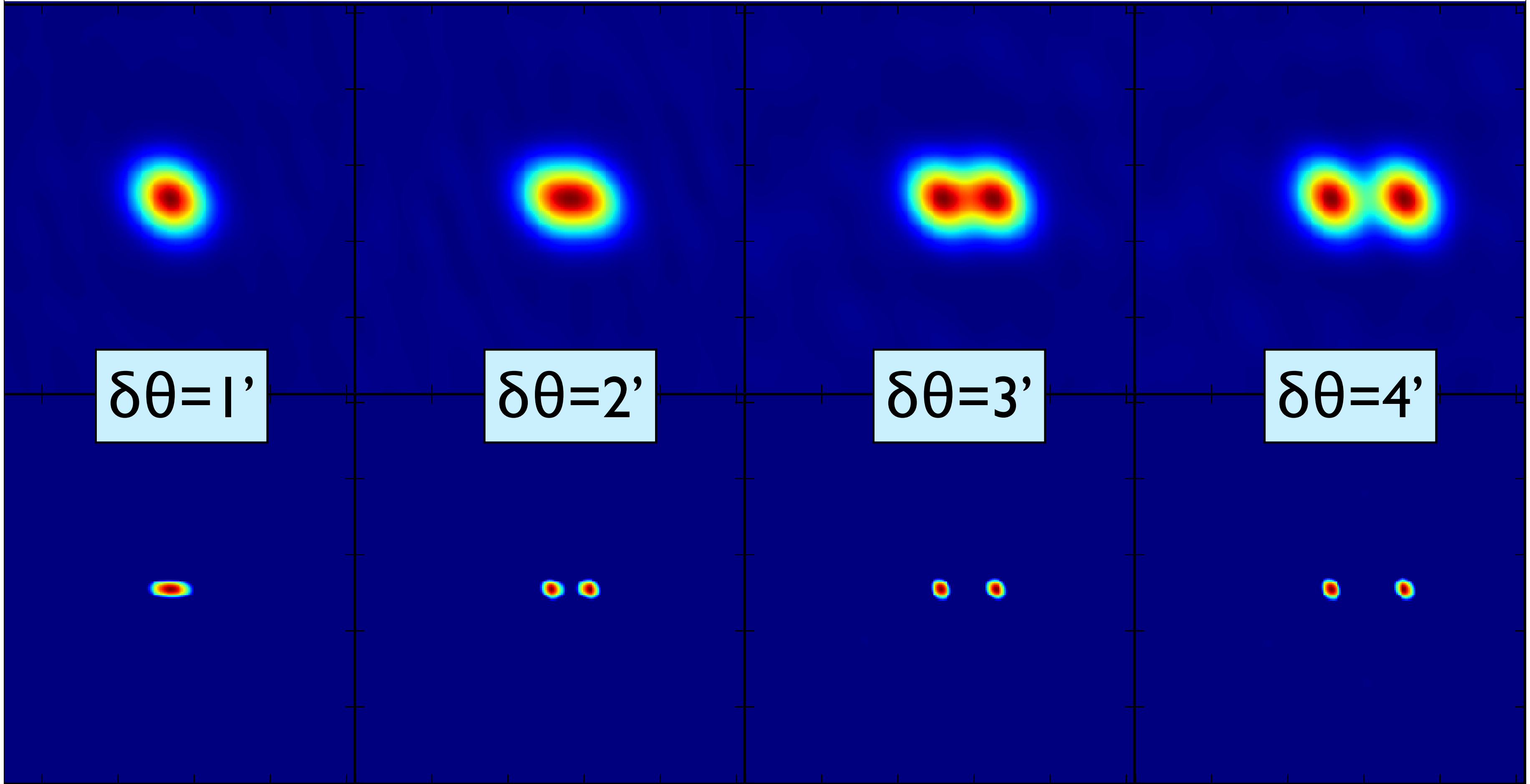
Sparse reconstruction

# Experiment #2: Angular separation

Noiseless data

**CLEAN**

CLEAN beam = 3.2’x2.5’



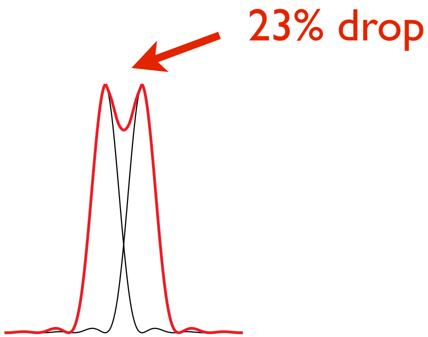

**Sparse recovery**

- Sparse Recovery resolution improved by at least 2 compared the CLEAN beam.
- Recovered « sub-beam » sources have correct fluxes (~2% error) & positions

# Experiment #2: Angular separation

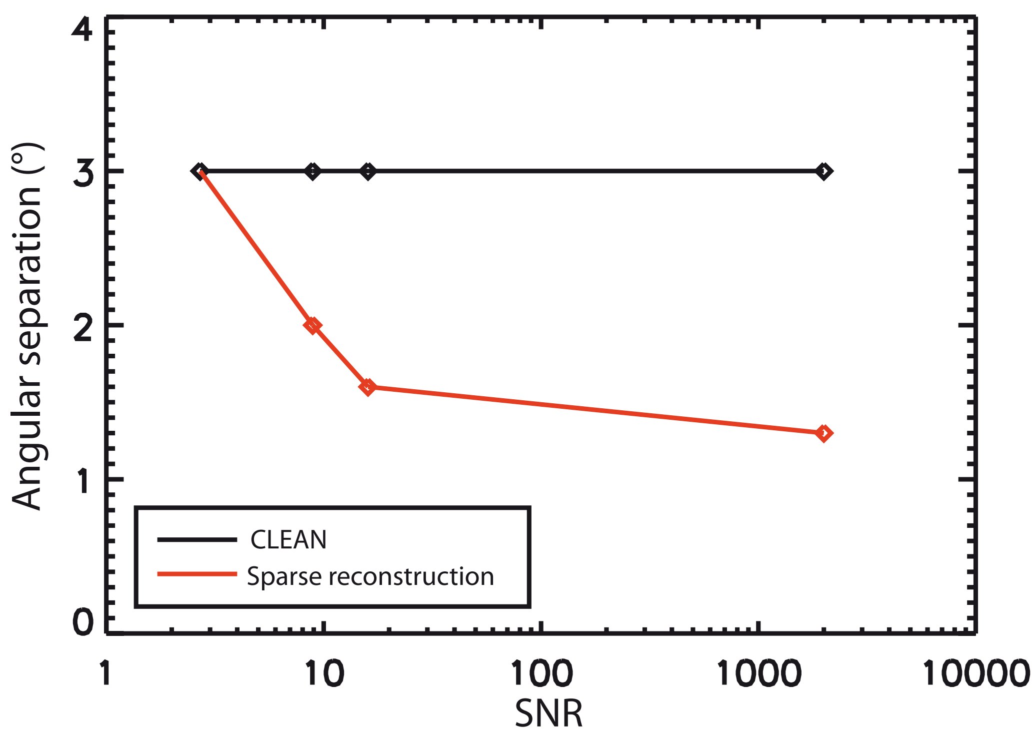
- On noisy data ➢ (rough) measurement of the source separability angle.

Rayleigh criterion




Separated sources
when decrease > 23%

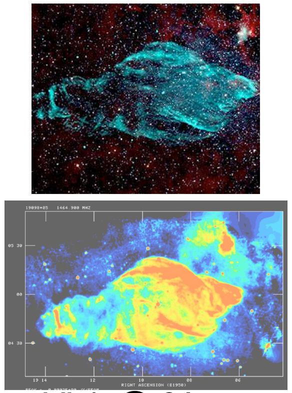
*Effective source separability vs. SNR*




==> Sparse reconstruction: angular separation **improved by 2 for SNR > 10**, and converges to CLEAN resolution at low SNR regimes.

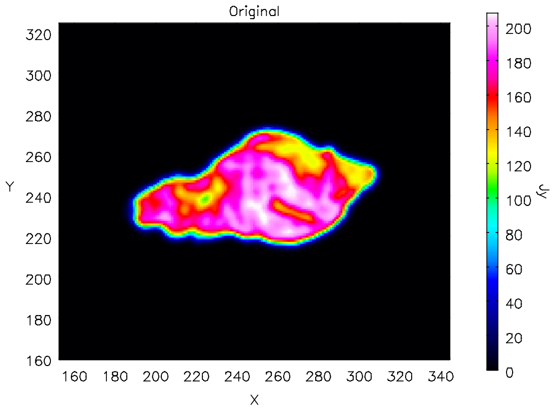
# Experiment #3: Extended source

- VLA 21-cm image of W50 + empty simulated LOFAR dataset
- Set to an arbitrary flux scale and converted to visibilities (AWimager)



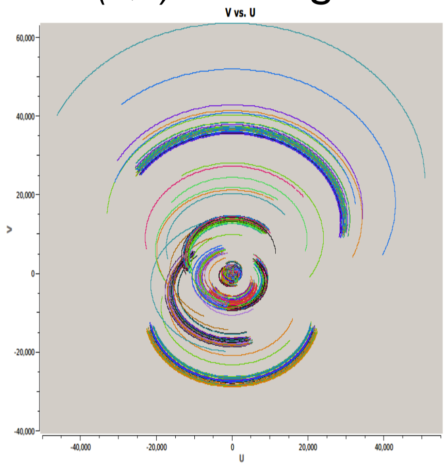
VLA @ 21 cm

*Model image*

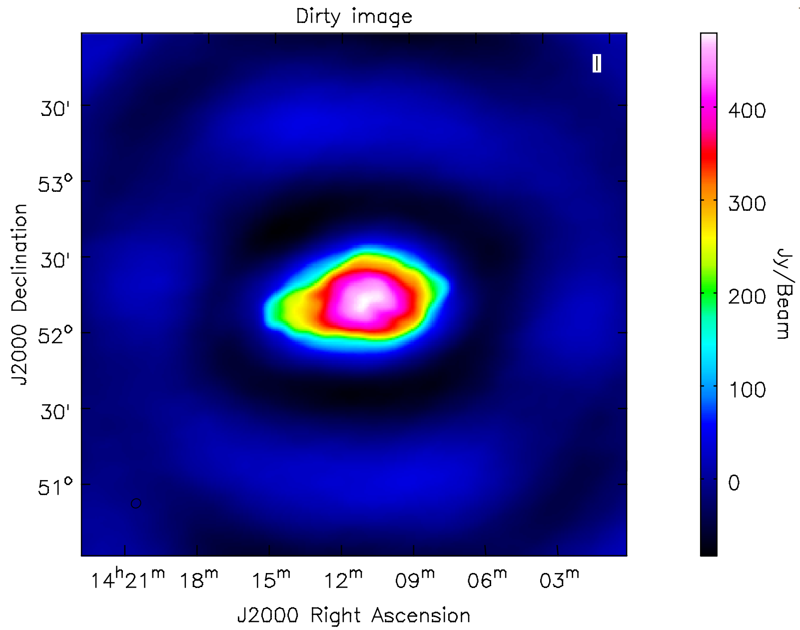



FFT
+
(u,v) Sampling

*(u,v) coverage*



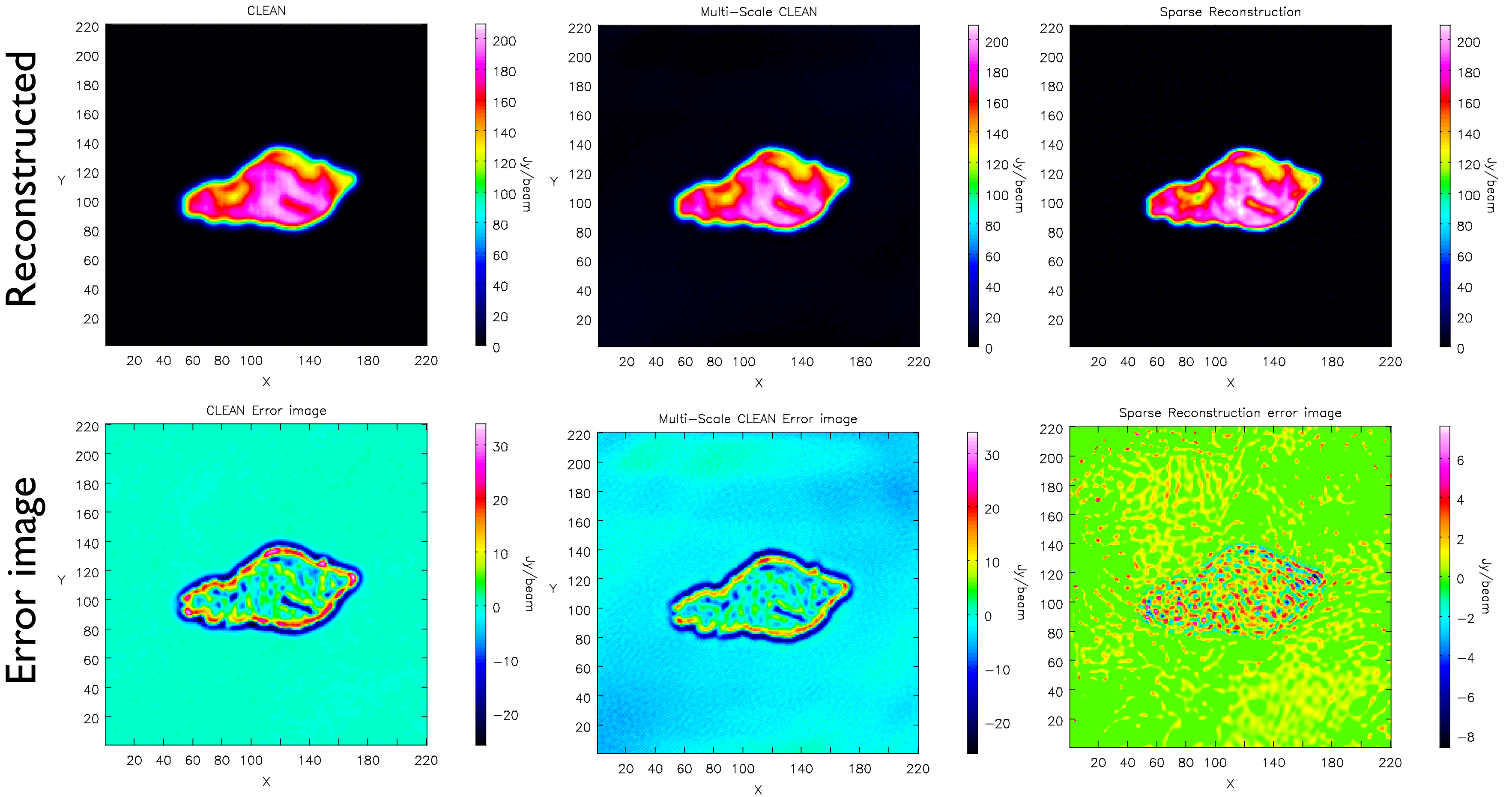

*Dirty image*



# Experiment #3: Extended source

- Using CLEAN, Multiscale CLEAN and Sparse reconstruction

CLEAN | Multiscale CLEAN | Sparse Reconstruction




RMS error = 3.50

RMS error = 3.28

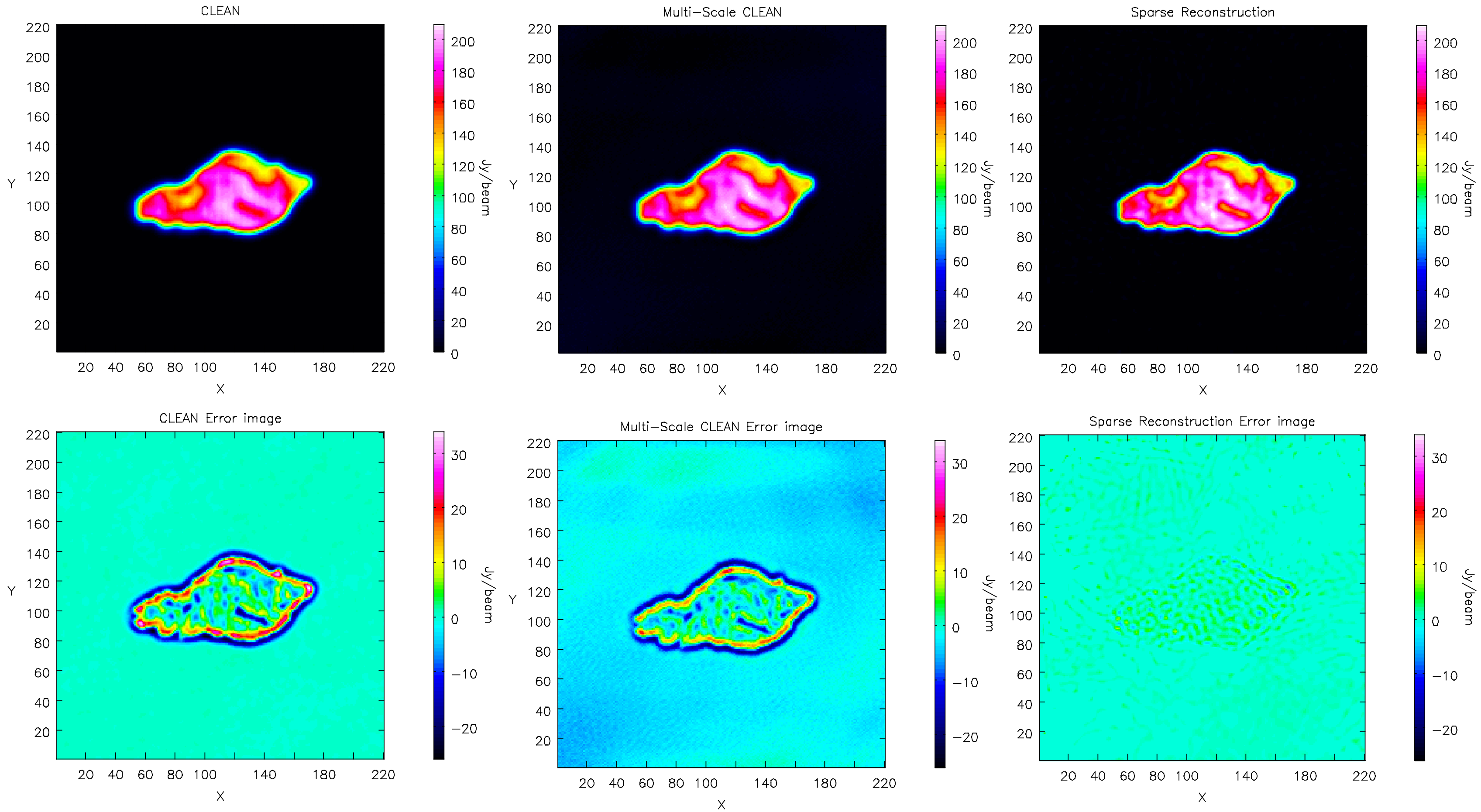
RMS error = 0.76

# Experiment #3: Extended source

- Using CLEAN, Multiscale CLEAN and Sparse reconstruction

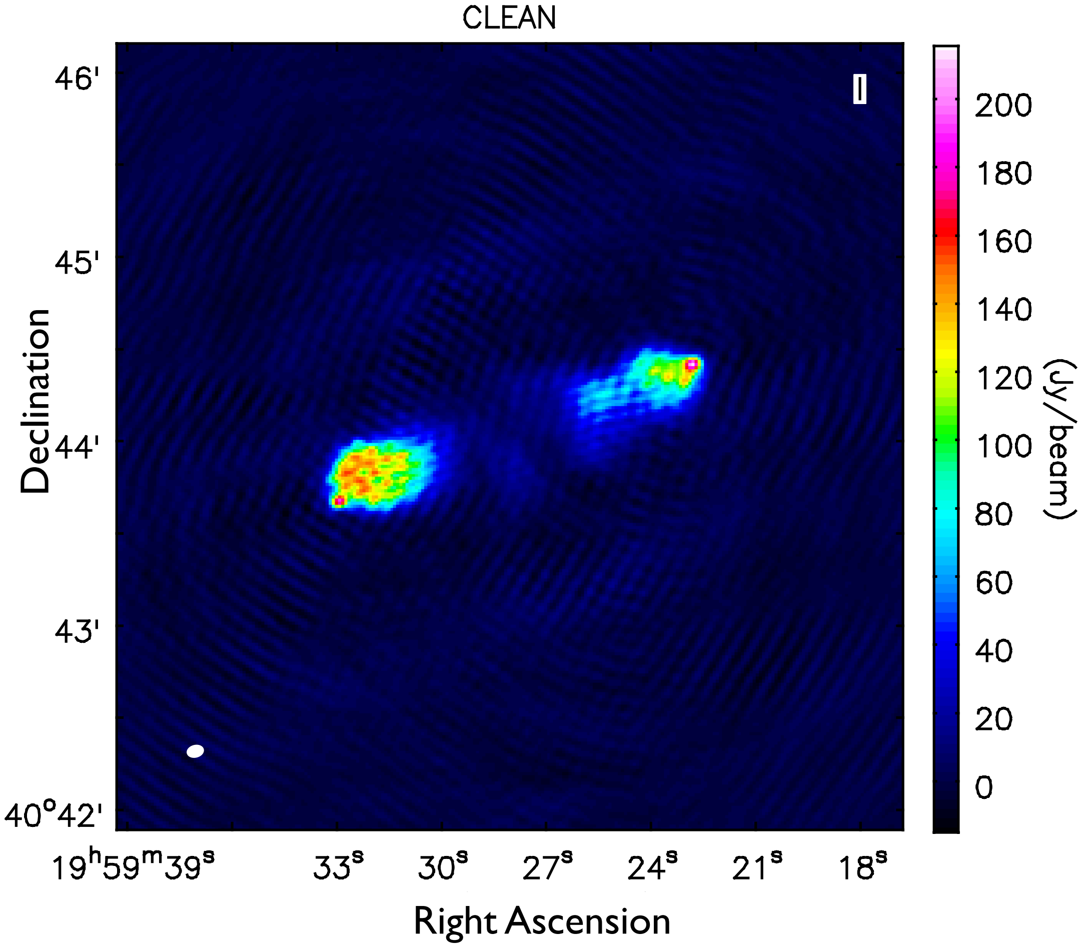
CLEAN | Multiscale CLEAN | Sparse Reconstruction

Reconstructed / Error image




RMS error = 3.50 | RMS error = 3.28 | RMS error = 0.76

# Experiment #4: Real data **Cygnus A**




F = 151 MHz - ΔF = 195 kHz

ΔT = 6 Hr

36 LOFAR Stations

*(dataset courtesy of John Mckean)*

## CLEAN

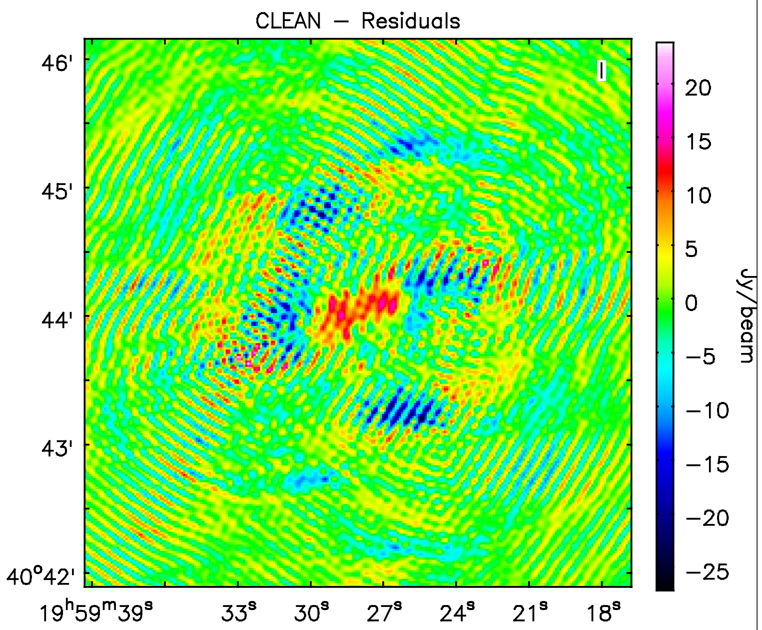
- Pixel = 1" size = 512 x 512
- Threshold = 0.5 mJy
- Weighting = super uniform

## Restored image

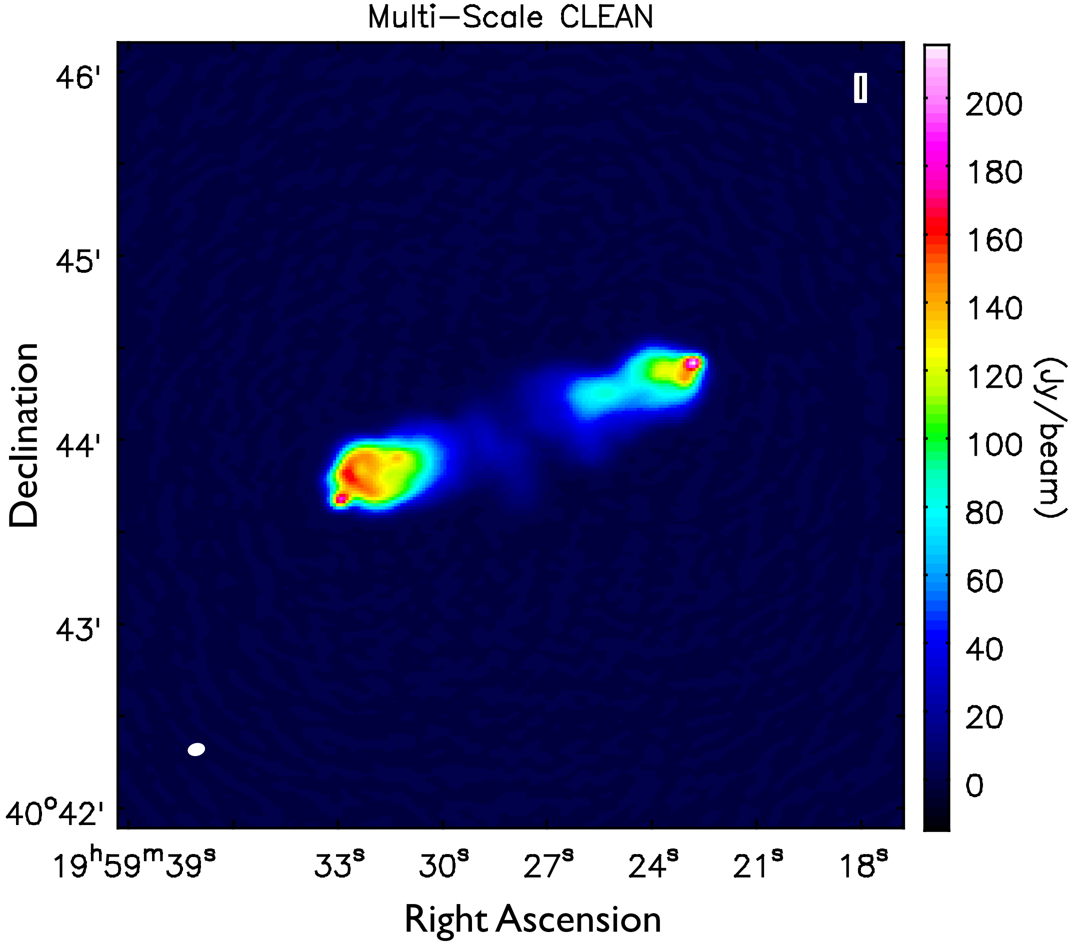
Total Flux density = 9393 Jy

## Residua

Residual std-dev = 2,65 Jy/beam



# Cygnus A




F = 151 MHz - ΔF = 195 kHz

ΔT = 6 Hr

36 LOFAR Stations

*(dataset courtesy of John Mckean)*

## Multi-Scale CLEAN

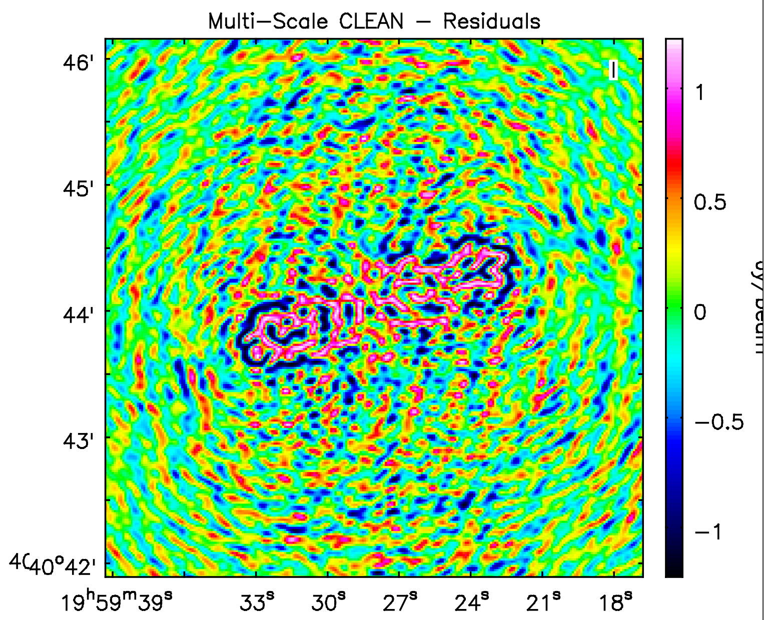
- Pixel = 1"  size = 512 x 512
- Threshold = 0.5 mJy
- Weighting = super uniform
- Scales = [0, 5, 10, 15, 20] pixels

## Restored image

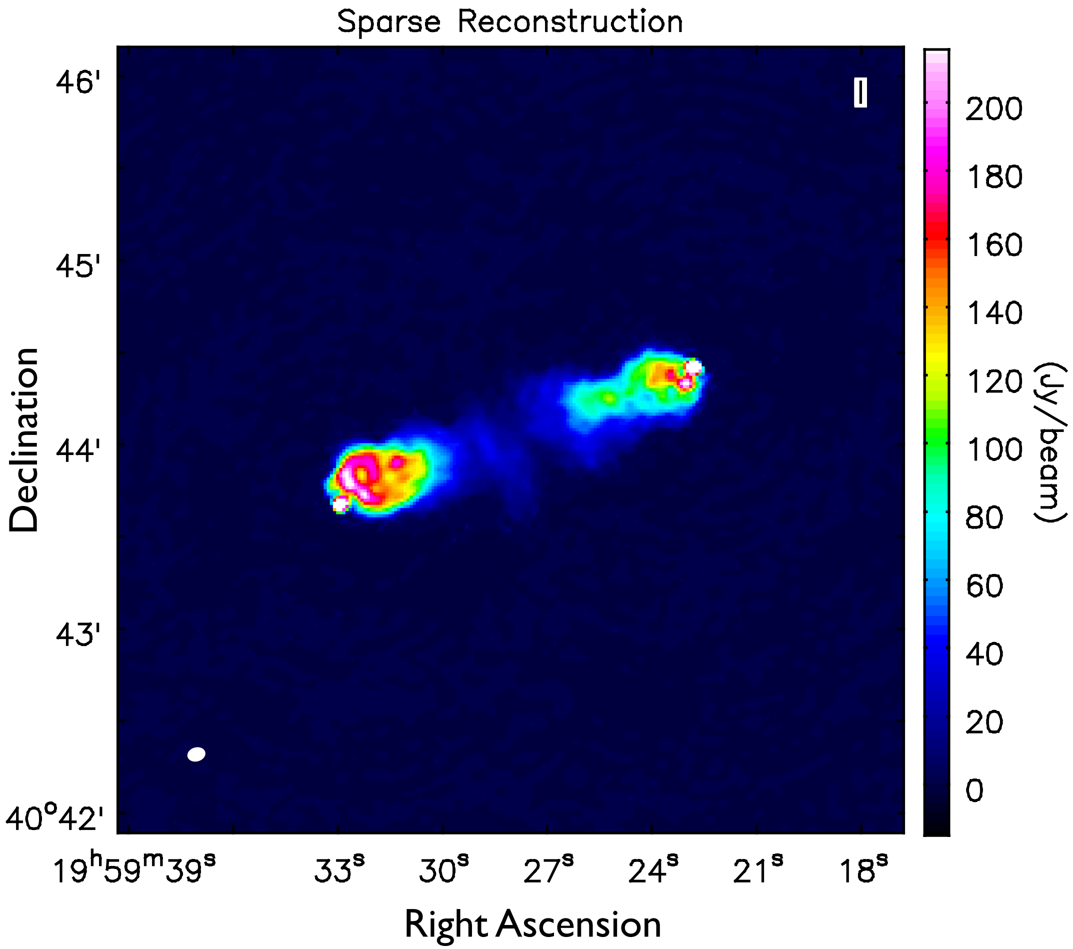
Total Flux density = 10553 Jy

## Residuals

Residual std-dev = 0,26 Jy/beam



# Cygnus A




F = 151 MHz - ΔF = 195 kHz

ΔT = 6 Hr

36 LOFAR Stations

*(dataset courtesy of John Mckean)*

## Sparse Reconstruction

- Pixel = 1''   size = 512 x 512
- Threshold = 0.5 mJy
- Weighting = super uniform
- Scales = 7 wavelets scales
- Minimization algorithm: FISTA

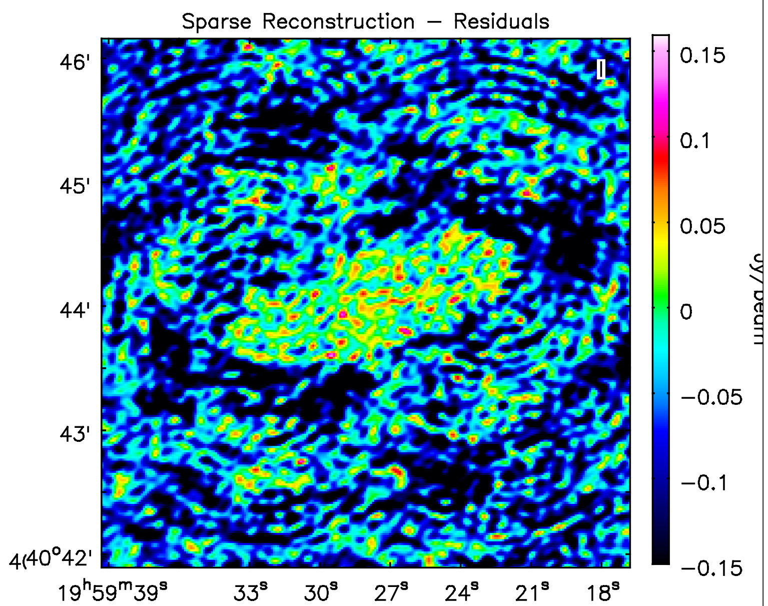
Fast Iterative Shrinkage-Thresholding Algorithm

## Restored image

Total Flux density = 10506 Jy

## Residuals

Residual std-dev = 0,05 Jy/beam



# Conclusions

- ✓ Sparse recovery is a totally new imaging method for LOFAR and other modern interferometers.

- ✓ Experimental results are good
  - Photometry: similar to CLEAN on point sources.
  - Resolution: improved by a factor 2 for SNR > 10.
  - Extended objects reconstruction much better than CLEAN and Multiscale CLEAN.
  - Improved image quality (RMS better by factor 5 compared to CLEAN)

- ✓ Will continue to develop (CLEAN has had 40 years)

- ✓ Sparsity also very efficient for **EoR signal extraction**: ***Chapman et al, MNRAS 429, Issue 1, p.165-176, arXiv:1209.4769, 2013.***

- ✓ Papers
  - H. Garsden, J-L. Starck, S. Corbel et al., "Compressed sensing imaging reconstruction for the LOFAR Radio Telescope", Proceedings of SPIE Vol. 8833 (2013)
  - Journal Paper in prep.